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A gert and gert III simulation approach to the evaluation of Markov process statistics with applications in the appraisal of newspaper subscription survivals.

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A GERT AND GERT III SIMULATION APPROACH TO THE EVALUATION OF MARKOV PROCESS STATISTICS WITH APPLICATIONS IN THE APPRAISAL OF NEWS-PAPER SUBSCRIPTION SURVIVALS

A Thesis
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ABSTRACT

The degree of applicability of GERT and GERT III simulation to the analysis and evaluation of the statistics obtained from Markov processes was studied by appraising the following Markov discrete time processes:

(a) The probability distributions of state occupancies.

(b) The transition and steady state probabilities.

(c) The mean state occupancies.

(d) The first passage time to a state.

(e) The moments of delay in transient Markov processes.

To prove the simplicity of analyzing Markov models, especially those consisting of large numbers of states, by GERT and GERT III simulation, they were utilized to study the duration of subscriptions of two daily newspapers.

The Windsor Star was originally chosen for this study. Since the constructed Markov models were found to consist of only two states: either subscribers or non-subscribers, the records of The Detroit Evening News were also studied in order to give more breadth to the investigation of GERT and GERT III to this type of problem. In the case of The Detroit Evening News, the Markov model was found to possess seven states.

The Markov models developed were tested for their validity, and analyzed employing GERT and GERT III simulation for the appraisal of the subscription duration for each of the two newspapers. The results for both The Windsor Star and The Detroit Evening News were compared.

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CHAPTER I

INTRODUCTION

Uncertainty, complexity and diversification have been continuing challenges to the understanding and control of the physical and social environment. In the development of logical structures to describe the majority of these phenomena, the model originated by A. A. Markov stands as a major accomplishment. Where previous contributors had modeled uncertainty as a sequence of independent trials, Markov saw the advantages of introducing the dependence of each trial on the result of its previous state, whenever it proved significant.

The analysis of Markov processes employing the matrix function approach proved to be quite complex, especially when it had to deal with repeated characteristic values of a transition matrix P. This problem would probably arise in polydesmic processes. Another technique was introduced in the study of Markov processes that employed the Z-transform. It is a simple and powerful technique, especially for finding a closed form for P^n (the transition probability matrix for n transitions). However, the latter technique has two main deficiencies: its algebra gets tedious when the number of states reaches five or six, and it is very ~~laborious~~ for computer programming.

Recently GERT (Graphical Evaluation and Review Technique) which allows the combination of the disciplines of flowgraph theory, moment generating functions and PERT (Project Evaluation Review Technique) was developed for the analysis of stochastic networks. Only a few researchers have employed GERT for the evaluation of Markov process, and it has been found to make the evaluation of certain Markov processes much simpler. More recently, GERT III which is a simulation language, was developed and was found to be extremely effective in analyzing a wider variety of stochastic network applications.

The aim of this research was to determine the applicability of GERT and GERT III to changes in the analysis and evaluation of Markov processes, especially those consisting of relatively large numbers of states. To prove the effectiveness of analyzing Markov models by GERT III, the subscription life of a daily newspaper was simulated. The basic concept of Markov processes required that the particular state a subscription was in at any given time determined the probability of the subscription's continuation or cancellation.

It was found that extremely high financial risks are frequently encountered in the newspaper industry. For example, unsold copies means a direct loss in revenue. Furthermore, the number of subscriptions of the paper may drop so rapidly that publication of the paper may cease, *The Windsor Star* was originally chosen for this study, but it was immediately found that the resulting Markov models contained only two states, either subscription or non-subscription. The records of *The Detroit Evening News* were then studied in order to give more breadth and versatility to this investigation. The model from the latter was found to contain seven states.

An interesting innovation of *The Detroit Evening News* is that it offers life insurance policies to its subscribers. These policies are restricted to subscribers of *The Detroit Evening News* and the subscribers appear to be more reluctant than others to drop their subscriptions lest they lose their insurance policy. This plan serves as a motivator to prolong the paper's average subscription life. *The Windsor Star* has no comparable incentive plan to stimulate or prolong subscriptions to it.

There are seven different subscription patterns for The Windsor Star - one for each of the following geographical areas:

(1) City Zone
(2) Retail Trading Zone
(3) County of Essex
(4) County of Kent
(5) County of Lambton
(6) County of Middlesex
(7) Southern Ontario (total)

It was assumed that the behavior pattern of subscribers to The Windsor Star could be realistically described by a stationary first order Markov process. The Chi-square test was applied to the hypothesis of a stationary chain in each case. The seven resulting Markov models were analyzed, using GERT III simulation. The hypothesis of a stationary chain was found to be acceptable for a level of significance of 10%.

Previous to this study, the staff of The Detroit Evening News ~~had~~ developed a rather limited Markov model. Advantage was taken of this work, which had been undertaken to provide information needed to settle a tax case.

The Study:

The study was conducted in the following sequence: a detailed study of the evaluation of the following Markov discrete time processes statistics employing GERT and GERT III simulation program.

(a) The probability distributions of state occupancies.

(b) The transition and steady state probabilities.

(c) The mean state occupancies.

(d) The first passage time to a state.

(e) The moments of delay in transient Markov processes.

After this study was completed, a definite understanding of the applicability of GERT and GERT III has been acquired to simulation program to the evaluation of Markov processes statistics, GERT III simulation was employed to analyze The Windsor Star and The Detroit Evening News cases in the following manner:

(a) The subscription life probability distribution for each of the seven pattern of The Windsor Star.

(b) The subscription life probability distribution of The Detroit Evening News for insured and uninsured subscribers were determined. Examinations were made on the effect of insurance in prolonging the subscription life.

(c) A general comparison between the subscription life of The Windsor Star and The Detroit Evening News.

CHAPTER II

LITERATURE SURVEY

2.1 Introduction:

The material covered in this chapter summarizes briefly the existing literature and is included here for ease of reference. The following survey consists of reviews and investigations of methods used. The bulk of literature is quite large, hence the present survey is selective, concentrating on those findings that seem best related to the scope of this research and those which can be adopted to analyze Markov processes.

2.2 Markov Processes:

The basic concept of Markov processes are those of "state" of a system and "state transitions". A system is occupying a state when it is completely described by the values of variables that define the state. A system makes state transitions when its describing variables change from values specified for one state to those specified for another.

If attention is focused on the state transitions of the system, and merely index the transition in time, then the system could be profitably thought of as a discrete-time process.

If the time between transitions is a random variable that is of interest, then the system is considered to be either a continuous-time Markov process or a semi-Markov process.

If the successive state occupancies of the system are governed by the transition probabilities of a Markov process, but the stay in any state is described by an integer-valued random variable that depends on the state presently occupied and on the state to which the next transition will be made, the system is a discrete time semi-Markov process.

A continuous-time semi-Markov process differs from the discrete time semi-Markov process in that the transitions of the system can occur after any positive, not necessarily integral, time spent in a state.

Kemeny and Snell (8) discussed evaluation of the first passage times and its variance for regular Markov chains.

A regular Markov chain, as defined by Kemeny and Snell "is one that has no transient sets, and has a single ergodic set with only one cyclic class."

2.3 <u>Transform Approach in Analyzing Markov Processes</u>:

Howard (6) introduced the concept of analyzing discrete time Markov processes, employing the geometric transform.

The suggested method of employing the geometric transform has proved to be more simple and powerful than other methods employing the matrix function approach, which proved it became quite complex when it had to deal with repeated characteristic values of the transition matrix P, a problem that is likely to arise in polydesmic processes.

Employing the geometric transform is a simple technique to finding a closed form expression for P^n.

Howard (6) stated that, in spite of the strength of the transform analysis, its algebra gets a little tedious when the number of states gets up around 5 or 6. Howard proved that the probability transformation presented by any Markov process is a linear system and that the powerful graphical techniques so useful in system analysis, can be applied to Markov processes. He stated that the restriction to discrete time processes is not essential. All the discrete time functions used in the analysis of linear systems which is applicable to Markovian processes could be continuous without introducing any real change in the results. The time function in the continuous case could be regarded as vectors with a nondenumerable number of components. The output vector from a linear system will be obtained by convolving the input vector with the impulse response vector. However, the convolution operation will be defined by a convolution integral rather than the convolution summation. The relevant transformation will be an exponential (or Laplace) transform instead of the geometric transform.

Howard also stated that the flowgraph relationships are identical for continuous time as of discrete time formulation. However, employing the z-transform to the analysis of discrete time Markov and semi-Markov processes is limited to functions that do not increase in magnitude faster than a geometric sequence. In the same manner, employing Laplace transform to the analysis of continuous time Markov and semi-Markov processes is limited to functions that do not increase in magnitude faster than an exponential sequence.

2.4 Flowgraph Techniques and Algebra:

D'Azzo and Constantine (2) and Howard (6) defined components of flowgraphs. They presented studies of flowgraph algebra. Howard (6) introduced the concepts of encysted and isolated systems, which could be of help in flowgraph reductions. Whitehouse (19), Howard (6) and D'Azzo and Constantine (2) discussed Mason's rule for flowgraph reduction, which is used extensively in this field.

Whitehouse (19) added the definition of the distributive nodes; they are nodes which have at most one transmittance terminate and at least two originates. He also added the definition of contributive nodes; these are nodes which have at least two transmittances terminates and at most one originate.

Whitehouse (19) submitted a computer program written in FORTRAN which solves flowgraph transmittances.

Pritsker and Kivat (1) developed GASP 11, which is a FORTRAN based simulation language, applicable to the study of probabilistic networks. GASP 11 has some deficiency, i.e., the optimal solutions obtained by simulation through the use of search techniques is beyond its scope.

Pritsker and Happ (16) developed GERT (Graphical Evaluation and Review Technique). It is a procedure for the analysis of stochastic networks composed of logical nodes and multiparameter branches (transmittances). It combines the disciplines of flowgraph theory, moment generating functions (MGF) and PERT to obtain a solution to stochastic problems.

Pritsker and Happ (16) claim that the analytic approach to solving GERT networks was still inadequate in some circumstances. These inadequacies led to the development of a simulation approach to the analysis of GERT networks. Pritsker developed a series of GERT simulation programs, the last of these being GERT III.

The GERT simulation program is a general purpose program for simulating networks. The program is written in FORTRAN IV. The input to the program is a description of the network in terms of its nodes and branches, along with control information for setting up simulation conditions. The output to the program consists of two main parts:

1) The final results of the simulation which contains information about nodes realizations such as, the mean time to realize a node and its standard deviation, the number of times a node was realized and the minimum and maximum times a node was realized.

2) Histograms which contain the frequency distribution of the number of times a node was realized vs the time of its realization.

In order to explore the applicability of GERT to the analysis of Markov and semi-Markov processes, a detailed study was conducted.

2.5 <u>Estimating the Parameters of the Markov Probability Models</u>:

Methods of estimating the parameters of zero and first order stationary Markov models from micro data are discussed in Bhat (13). Lee, Judge and Takayama(11) indicated that micro data are not aways available and hence they introduced two new methods of estimating the transition probability matrices for Markov processes from aggregate (macro) data. These two methods are: the restricted least square estimator (RLS), and the minimum absolute deviation (MAD). Lee, Judge and Zellner (10) later on expanded their research in the estimation of the transition probability matrices for Markov processes from aggregate data in reference (10). They introduced other methods of estimation, such as the minimum Chi-square estimator, the macro maximum likelihood estimator (MLE) and the generalized least square estimator (GLS). They also wrote a computer program which can be employed to estimate the transition probability matrices of stationary first order Markov processes, using any of the mentioned estimators.

CHAPTER III

GERT and GERT III Analysis of Markov Processes

3.1 Analysis of Discrete Time Markov-Process

3.1.1 Introduction:

It has been found that the best suitable representation of a Markov state employing GERT elements will usually be a node with an EXCLUSIVE - OR input and PROBABILISTIC output.

For a discrete time Markov process, the time between transitions is constant and usually considered unity. The MGF of such a distribution is:

$$M(s) = e^{s}$$

3.1.2 Probability Distribution of State Occupancies:

This is an adaptation of Howard's (6) concept of the multiple transition classes. His concept is that each of the N^2 transitions that an N state process can make, can be divided into $M \leq N^2$ mutually exclusive and collectively exhaustive classes type 1, type 2, ..., type M. The transition Matrix P would then be decomposed in the form:

$$P = {}_1P + {}_2P + \ldots + {}_MP$$

Then $\Phi(K_1, K_2, \ldots, K_M \backslash n)$; the probability that each number of transitions K of each type would be made, is the probability distribution to be sought.

To evaluate values of $\Phi_{ij}(K_1, K_2, \ldots, K_M/n)$ employing GERT 111 simulation program, a numerical example is considered.

3.1.2a <u>A Numerical Example</u>:

Assuming a two state Markov process, the transition probability Matrix P is:

$$P = \begin{vmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{vmatrix}$$

assuming that only values of $\Phi_{11}(K/n)$ are of interest. The problem can be represented by GERT III elements, as shown in Fig. 1.3.

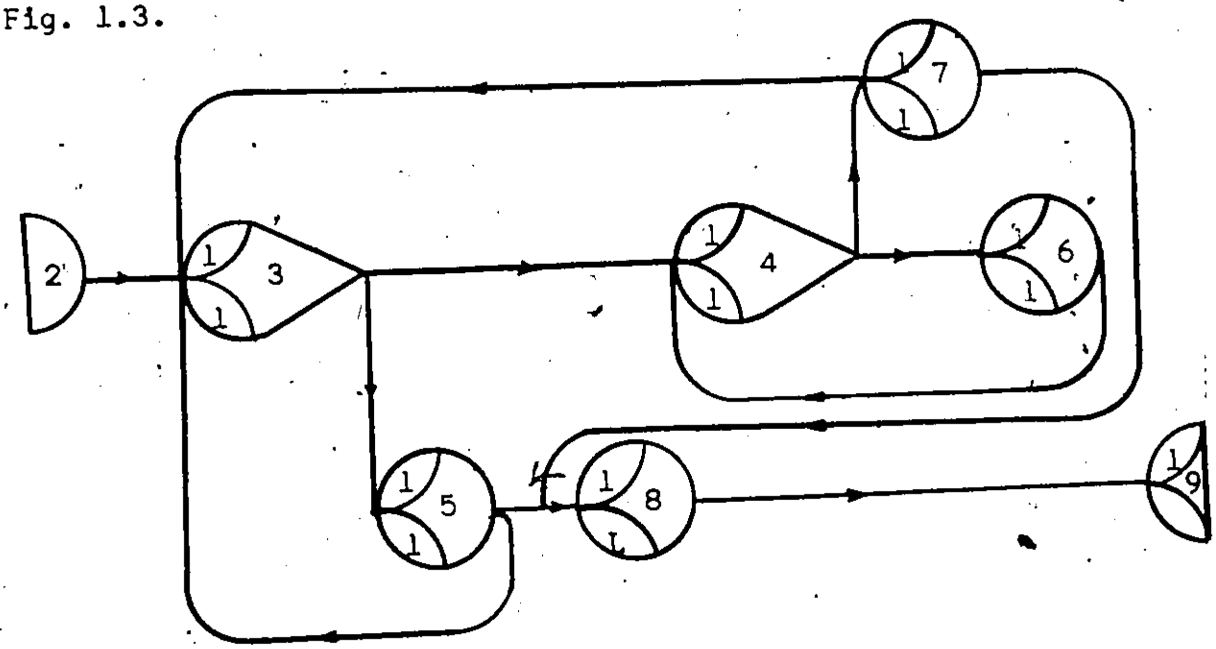

Fig. 1.3 GERT 111 adapted to represent a two-state Markov process.

Node 3 represents state one, node 4 represents state two and node 8 controls the number of state one occupancies in each run. L will be made equal to 1, 2, ..., 10 in ten runs; i.e., statistics on state one are to be collected for K = 1, 2, ..., 10.

The GERT 111 computer programs are presented in Fig.1(A). The simulation output in Fig.2(A) a shows that, out of a thousand simulations, it took one transition to realize node 9, 792 times, two transitions to realize node 9 60 times, etc...

This can be interpreted as the probability that the process makes a transition from state one to state one in one transition, $\Phi_{11}(2/1) = 792/1000 = 0.792$

Similarly: $\Phi_{11}(2/2) = 0.060$

Values of $\Phi_{11}(3/n)$ can be found from Fig.2(A) b, e.g.,

$$\Phi_{11}(3/1) = 0 \qquad \Phi_{11}(3/2) = 639/1000 = 0.639.$$

Generally speaking, values of $\Phi_{11}(K/n)$ for K = 0, 1, ..., 10 may be evaluated from Fig.2(A). They are summarized in Table 1.3

Howard (6) derived an equation to evaluate $\Phi_{11}(K/n)$ employing the geometric transformation. For the same numerical example, Howard's equation is:

$$\Phi_{11}(K/n) = 0.7\,\Phi_{11}(K/n-1) + 0.8\,\Phi_{11}(K-1/n-1) - 0.5\,\Phi_{11}(K-1/n-2) + \delta(K-1)\delta(n) - 0.7\delta(K-1)\delta(n-1) \quad --- \quad 1.3$$

K \ n	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0
2	0	0.792	0.060	0.037	0.035	0.023	0.016	0.012	0.006	0.011	0.002
3	0	0	0.639	0.109	0.055	0.046	0.041	0.025	0.020	0.027	0.012
4	0	0	0	0.490	0.114	0.104	0.071	0.042	0.041	0.034	0.027
5	0	0	0	0	0.425	0.141	0.098	0.068	0.059	0.054	0.032
6	0	0	0	0	0	0.315	0.147	0.091	0.084	0.065	0.072
7	0	0	0	0	0	0	0.255	0.145	0.093	0.081	0.080
8	0	0	0	0	0	0	0	0.200	0.110	0.093	0.093
9	0	0	0	0	0	0	0	0	0.163	0.106	0.092
10	0	0	0	0	0	0	0	0	0	0.125	0.087
11	0	0	0	0	0	0	0	0	0	0	0.100
$\sum_{K=0}^{\infty}\Phi_{11}(K/n)$	1	0.792	0.699	0.636	0.629	0.629	0.628	0.583	0.576	0.596	0.597

Table 1.3 Values of $\Phi_{11}(K/n)$ evaluated by GERT 111 simulation.

Some values of $\Phi_{11}(K/n)$ evaluated by solving equation 1.3 recursively are presented in Table 2.3:

K \ n	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	0	0.8	0.06	0.042
3	0	0	0.64	0.096
4	0	0	0	0.512

$\sum_{K=0}^{\infty} \Phi_{11}(K/n) = 1$

Table 2.3 - Solution by recursion for $\Phi_{11}(K/n)$

To compare the results of GERT III to those obtained by Howard, the evaluated values of $\Phi_{11}(1/n)$, $\Phi_{11}(2/n)$ and $\Phi_{11}(3/n)$ were compared as an example. For $\Phi_{11}(1/n)$ and $\Phi_{11}(2/n)$, the evaluated values were found to be identical to their expected values.

For $\Phi_{11}(3/n)$, it was necessary to test the hypothesis that the evaluated values of $\Phi_{11}(3/n)$ are acceptable estimates for their expected values. The Chi-square test was used with a resulting Chi-square value of 6.00, which is below the critical value 6.25 associated with the 10% significance level for three degrees of freedom. Thus the hypothesis that the values evaluated by GERT III for $\Phi_{11}(3/n)$ are acceptable estimates of their expected values.

3.1.3 Transition and Steady State Probabilities:

It would have been noticed in the previous section that the sum of $\Phi_{11}(K/n)$ over K is merely $\Phi_{11}(n)$, the n-step transition probability from state one to state one, thus:

$$\Phi_{11}(n) = \sum_{K=0}^{\infty} \Phi_{11}(K/n)$$

considering the numerical example in section 3.1.2a, the values for $\Phi_{11}(n)$ evaluated by GERT III simulation are shown at the bottom of Table 1.3.

The theoretical values are shown at the bottom row of Table 2.3. To compare both values of $\Phi_{11}(n)$, the Chi-square test of hypothesis was conducted. The Chi-square value resulting was 0.38, which is well below the critical value of 6.25 associated with a 10% level of significance for three degrees of freedom. Consequently, the results evaluated by GERT III was shown to be acceptable within a reasonable range of accuracy.

It should also be noticed that $\Phi_{11}(10) = 0.597$, which is very close to the theoretical value for the steady state probability of being in state one: $\Phi_1 = 0.600$.

3.1.4 Mean State Occupancies:

The state occupancy random variable $v_{ij}(n)$ is defined as the number of times state j is entered through time n, given that the system started in state i at time zero.

Considering the numerical example in section 3.1.2a from the simulation output Fig. 2(A)a, statistics collected on node 3 (which represents state one) shows that it was realized 1000 times at time zero, i.e., the mean number of times node 3 was realized at time zero was 1000/1000=1.

Similarly, Fig.2(A)b shows that node 9 was realized 1819 times after one transition, i.e., the mean number of times node 3 was realized after one transition is 1819/1000=1.819, or, in other words, the mean number of times state one was entered through one transition, given that the process started in state one is 1.819. The results of the simulation are summarized in Table 3.3.

The Chi-square value resulting was 1.98, which is well below the critical value 14.68 which is associated with the 10% level of significance for nine degrees of freedom. Thus the evaluated values of $\bar{V}_{11}(n)$ are shown to be acceptable estimates of their expected values.

n	0	1	2	3	4	5	6	7	8	9
$\bar{V}_{11}(n)$	1	1.819	2.452	3.180	3.759	4.366	4.955	5.620	6.116	6.805

Table 3.3 Values of $\bar{V}_{11}(n)$ evaluated by GERT 111.

Values of $\bar{V}_{11}(n)$ obtained using analytical techniques are shown in Table 4.3.

n	0	1	2	3	4	5	6	7	8	9
$\bar{V}_{11}(n)$	1	1.8	2.5	3.15	3.775	4.387	4.994	5.597	6.119	6.800

Table 4.3 Theoretical values of $\bar{V}_{11}(n)$.

It can be observed that $\bar{V}_{11}(n)$ values evaluated by GERT 111 simulation and its theoretical values are very close.

3.1.5 The First Passage Time:

The first passage time is defined as the number of transitions it will take to reach state j for the first time if the system is in state i at time zero. Applying this definition to the GERT network , in example 3.1.2.a; since the process starts in node 3, the first passage time to node 3 will be the mean time elapsed to realize node 9 only once.

From the GERT 111 output, the first passage time to node 3 (which represents state one) $\bar{\theta}_{11}$ and its standard deviation $\overset{s}{\theta}_{11}$ are:

$$\bar{\theta}_{11} = 1.6950 \qquad \overset{s}{\theta}_{11} = 1.7587$$

Theoretical values of $\bar{\theta}_{11}$ and $\overset{s}{\theta}_{11}$ are:

$$\bar{\theta}_{11} = 1.6667 \qquad \overset{s}{\theta}_{11} = 1.82$$

which shows that values of $\bar{\theta}_{11}$ and $\overset{s}{\theta}_{11}$ evaluated by GERT 111 simulation can be accepted within a reasonable accuracy.

3.1.6 Transient Markov Processes:

The Markov processes containing transient states in which the time or number of transitions required to enter one of the recurrent states from each transient state is the random variable of central importance. An example of the transition probability Matrix P of a transient Markov three state process is:

$$P = \begin{vmatrix} P_{11} & P_{12} & 0 \\ 0 & P_{22} & P_{23} \\ 0 & 0 & 1 \end{vmatrix}$$

where state three is a trapping state; it is also the only recurrent state.

3.1.6.a Representation by GERT:

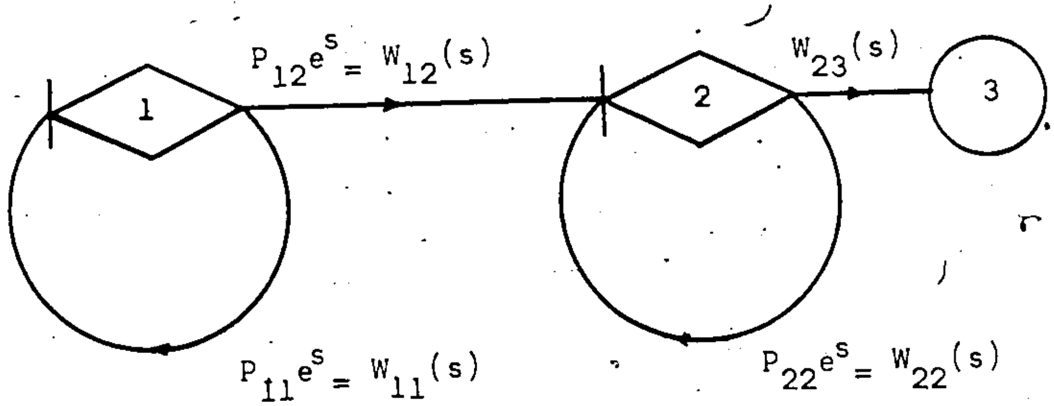

Fig. 2.3 GERT representation of the transient process.

3.1.6.b Moments of Delay of the Transient Process:

In many applications, the moments of the number of transitions to enter the trapping state or delay are of immediate interest.

Applying GERT analysis to example 3.2.b, assuming the process starts at state one:

$$W_e(s) = \frac{W_{12}(s)\ W_{23}(s)}{(1-W_{11}(s))\ (1-W_{22}(s))}$$

The probability of realizing the network P_e is:

$$P_e = W_e(s)\Big|_{s=0}$$

The MGF of the time to realize the network is:

$$M_e(s) = \frac{W_e(s)}{W_e(s)\big|_{s=0}}$$

The first moment $\bar{v}$ is:

$$\bar{v} = \left.\frac{dM_e(s)}{ds}\right|_{s=0}$$

The second moment $\overline{v^2}$ is:

$$\overline{v^2} = \left.\frac{d^2M_e(s)}{d^2s}\right|_{s=0}$$

And the variance $\overset{v}{v}$ is:

$$\overset{v}{v} = \overline{v^2} - \bar{v}^2 ,$$

3.1.6.c Numerical Example:

Assuming the transition probability Matrix P in the transient Markov process described in the example section 3.1.6.b is

$$P = \begin{vmatrix} 0.8 & 0.2 & 0 \\ 0 & 0.7 & 0.3 \\ 0 & 0 & 1 \end{vmatrix}$$

The W function is thus:

$$W_e(s) = \frac{0.06e^{2s}}{(1-0.8e^s)(1-0.7e^s)}$$

$$P_e = W_e(s)\Big|_{s=0} = 1$$

$$M_e(s) = \frac{W_e(s)}{W_e(s)\Big|_{s=0}} = W_e(s)$$

The mean number of transitions (time) before getting into the trapping state:

$$\bar{v} = \left.\frac{dM_e(s)}{ds}\right|_{s=0} = 8\ 1/3$$

The second moment:

$$\overline{v^2} = \frac{d^2 M_e(s)}{ds^2}\Bigg|_{s=0} = 875/9$$

The variance:

$$\overset{v}{v} = \overline{v^2} - \bar{v}^2 = 875/9 - 625/9 = 250/9$$

The standard deviation $\overset{s}{v}$:

$$\overset{s}{v} = \sqrt{\overset{v}{v}} = 5.3$$

3.1.6.d <u>Moments of Delay in a Transient State</u>:

The expected delay is the expected number of occupancies. State j is occupied when the system is started in state i, before the system gets into the trapping state.

Considering example 3.1.6.a, to determine the expected number of occupancies state one is occupied when the system is started in state one, the problem is represented by GERT, as shown in Fig. 3.3.

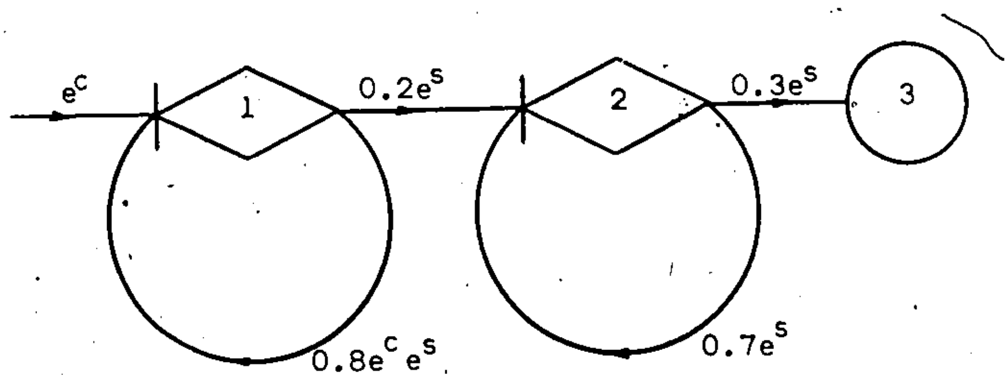

Fig. 3.3 GERT network adapted to evaluate the expected delay.

Each branch entering node 1 is tagged with e^c. The reason for this is that the sum of the transitions of all the paths leading to node 1 is equivalent to the number of times this node is realized; thus:

$$W(s,c)=M(s,c)= \frac{0.06e^c e^{2s}}{(1-0.8e^c e^s)(1-0.7e^s)}$$

$$M(s,c)\Big|_{s=0} = M(c) = \frac{0.2e^c}{(1-0.8e^c)}$$

The expected number of occupancies $\bar{v}_{11}$ is thus:

$$\bar{v}_{11} = \frac{dM(c)}{dc}\Bigg|_{c=0} = \frac{0.2e^c}{(1-0.8e^c)} + \frac{0.2x0.8e^{2c}}{(1-0.8e^c)^2}\Bigg|_{c=0} = 5$$

The second moment of delay $\overline{v_{11}^2}$ is:

$$\frac{d^2M(c)}{dc^2}\Bigg|_{c=0} = \frac{0.2e^c}{(1.08e^c)} + \frac{3x0.2x0.8e^{2c}}{(1-0.8e^c)^2} + \frac{0.2x0.8x0.8x2}{(1-0.8e^c)^3}\Bigg|_{c=0} = 45$$

The variance of delay $\check{v}_{11}$

$$\check{v}_{11} = \overline{v_{11}^2} - \bar{v}_{11}^2 = 20$$

To determine the expected number of occupancies state two is occupied with the system is started in state one, the problem is represented by GERT as shown in Fig. 4.3.

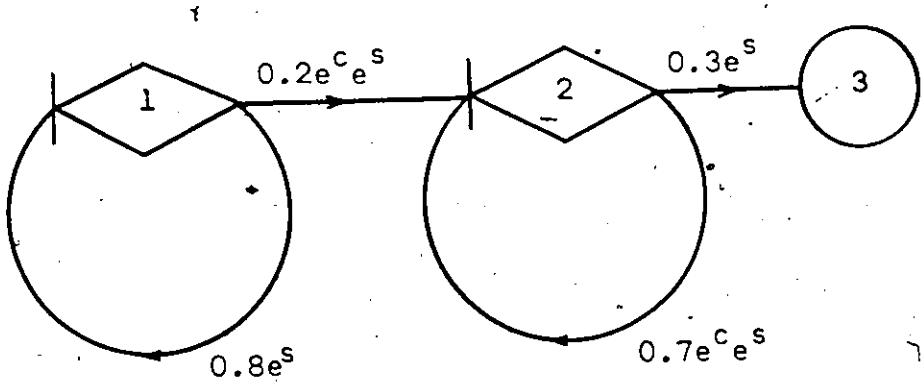

Fig. 4.3 GERT network adapted to find the expected delay in state two.

Each branch leading to state two is tagged with e^c.

$$W(s,c) = M(s,c) = \frac{0.06e^{c}e^{2s}}{(1-0.8e^{s})(1-0.7e^{c}e^{s})}$$

$$M(s,c)\Big|_{s=0} = M(c) = \frac{0.3e^{c}}{(1-0.7e^{c})}$$

The expected number of occupancies $\bar{v}_{22}$ is therefore:

$$V_{22} = \frac{dM(c)}{dc}\Big|_{c=0} = \frac{0.3e^{c}}{(1-0.7e^{c})} + \frac{0.3\text{x}0.7e^{2c}}{(1-0.7e^{c})^{2}}\Big|_{c=0} = 10/3$$

The second moment of delay $\overline{v_{22}^2}$ is:

$$\frac{d^2M(c)}{dc^2}\Big|_{c=0} = \frac{0.3e^{c}}{(1-0.7e^{c})} + \frac{3\text{x}0.3\text{x}0.7e^{2c}}{(1-0.7e^{c})^{2}} + \frac{0.3\text{x}0.7\text{x}0.7\text{x}2}{(1-0.7e^{c})^{3}}\Big|_{c=0} = 170/9$$

The variance of delay is:

$$\overset{v}{v}_{22} = \overline{v^2_{22}} - \bar{v}^2_{22} = 70/9$$

If the system was started in state two instead of state one, the GERT representation of the problem may be as shown in Fig. 5.3.

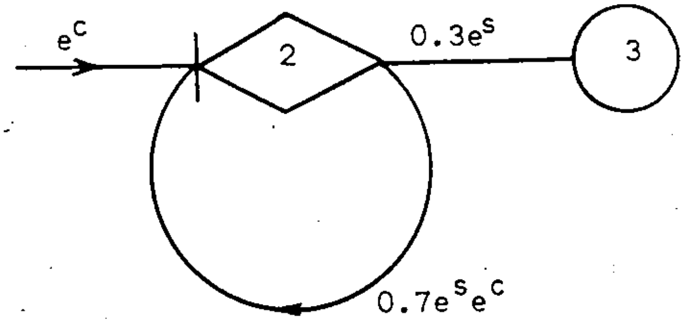

Fig. 5.3 The GERT network representation of a transient process.

The moments of delay in state two can be found in the same manner as before.

3.1.6.e GERT 111 Simulation to Evaluate the Transition Probabilities and the Moments of Delay:

Considering the numerical example in section 3.1.6.a, a GERT 111 simulation can be written to evaluate, 1) P(n), the probability that the system gets into trapping state three after n transitions if it is started in state one at time zero. 2) $\bar{v}$, the expected number of transitions the process will make until it gets into the trapping state three, and its standard deviation $\overset{s}{v}$ if the system was started in state one at time zero.

The GERT 111 network is shown in Fig. 6.3, where node 7 represents state three, the trapping state.

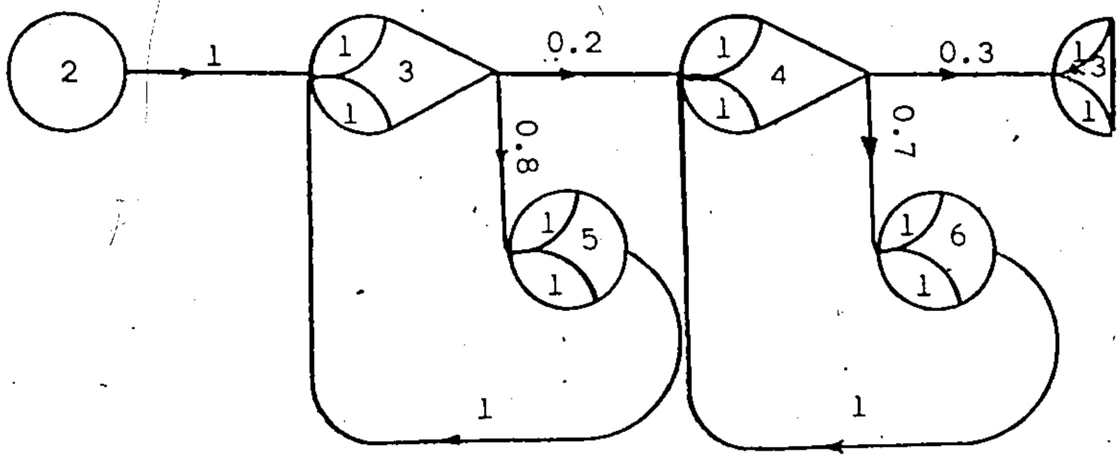

Fig. 6.3 GERT 111 network to represent a Markov process with one trapping state.

The GERT 111 program is presented in Fig. 3(A), the network is simulated 1000 times. The output (Fig. 4(A)) shows that node 7 (representing the trapping state) was realized zero times in zero and 1 transition. It was realized 52 times in 2 transitions, etc... Thus, the probability of the process to get into the trapping state in zero and one transition is zero. The probability after two transitions is 52/1000 = 0.052 and so on. These results are summarized in Table 5.3.

n	0	1	2	3	4	5	6
P(n)	0	0	0.061	0.079	0.111	0.106	0.091

Table 5.3 Values of P(n) evaluated by GERT 111 simulation.

Values of P(n) obtained by mathematical evaluation are shown in Table 5.4:

n	0	1	2	3	4	5	6
P(n)	0	0	0.06	0.09	0.1014	0.1017	0.0958

Table 5.4 - Actual values of P(n).

Comparing the values of P(n) that were evaluated by GERT III with their expected values by conducting the Chi-square test of hypothesis. The Chi-square value was 2.67. This value is well below the critical value of 10.64 associated with the 10% significance level for six-degrees of freedom. Thus the evaluated values of P(n) were found to be acceptable estimates for their expected values.

From Fig.3(A) it can be seen that the mean number of transitions the process makes until it gets into the trapping state (node 7) is: $\bar{V} = 8.4080$ and the standard deviation is: $V^s = 5.4354$ while values of $\bar{V}$ and V^s obtained by mathematical evaluation are:

$$\bar{V} = 8.333 \text{ and } V^s = 5.3$$

3.1.7 <u>Transient Processes with Several Trapping States</u>:

Occasionally the construction of models requires the consideration of a transient process that can run into a number of different trapping states. These transient processes are slightly different than those discussed in the previous section, since the probability "runs out of several drains" rather than just one.

An example of a transition matrix P of a four-state process, in which two states of the four are trapping states is:

$$P = \begin{vmatrix} 0 & P_{12} & P_{13} & 0 \\ P_{21} & 0 & 0 & P_{24} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

where state 3 and state 4 are the two trapping states. This example could be represented by GERT elements, as shown in Fig. 7.3:

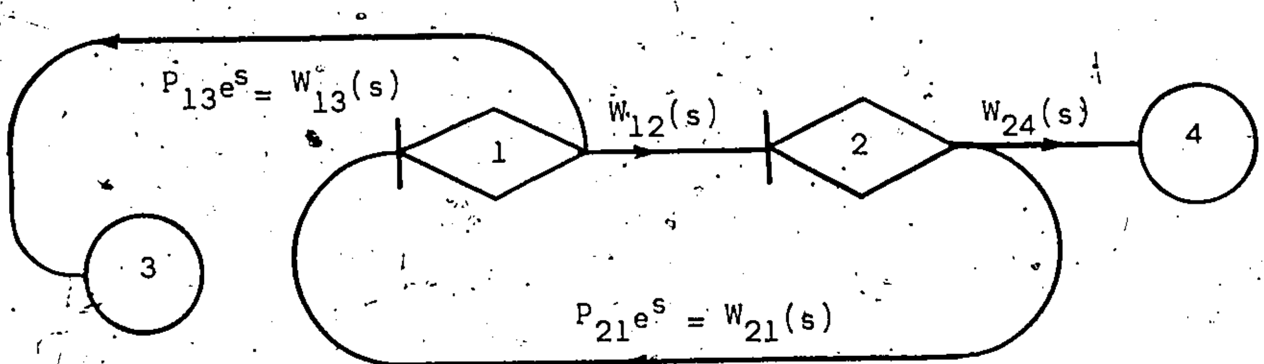

Fig. 7.3 GERT network adapted to represent a four-state transient process with two trapping states.

3.1.7.a <u>The Expected Delay in the Process, given the Final State</u>:

Considering the four-state example described in the previous section, it is assumed that the process will end in the trapping state 3, assuming further that the process has an equally likely chance to start in either state one or two.

A GERT approach may be used to evaluate the expected delay under such assumptions. A GERT representation for such an example is shown in Fig. 8.3.

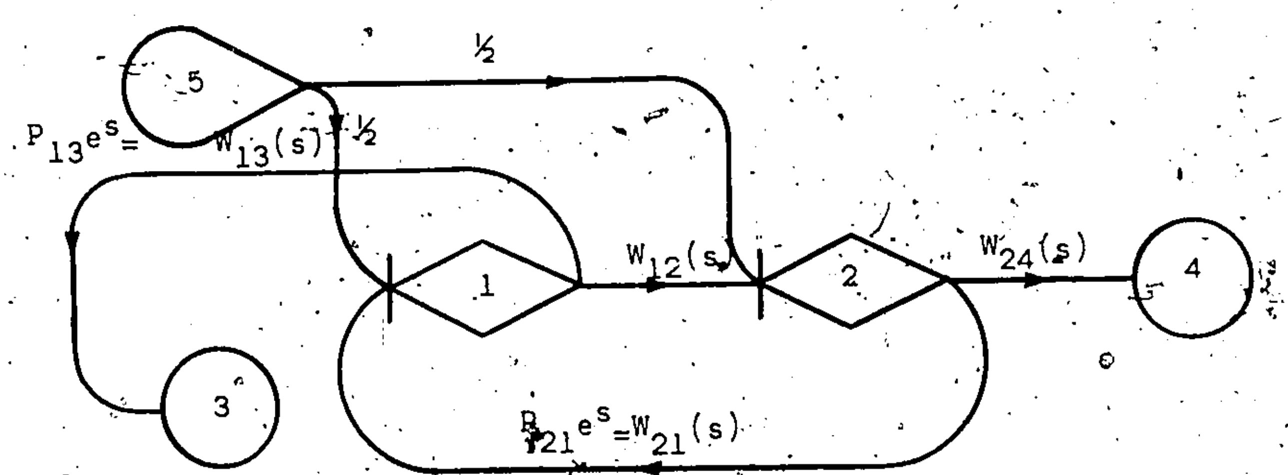

Fig. 8.3 GERT network adapted to represent the case of a random start in a transient process.

The W function is:

$$W_e(s) = \frac{\tfrac{1}{2}\,W_{21}(s)\,W_{13}(s) + \tfrac{1}{2}\,W_{13}(s)}{1 - W_{21}(s)\,W_{12}(s)}$$

The probability of getting into the trapping state three is thus:

$$P_e = W_e(s)\Big|_{s=0}$$

The MGF of the time to get to state three is thus:

$$M_e(s) = \frac{W_e(s)}{W_e(s)\Big|_{s=0}}$$

and the expected delay before the process is trapped in state three, $\bar{V}\Big|_3$ is thus:

$$\bar{V}\Big|_3 = \frac{dM_e(s)}{ds}\Big|_{s=0}$$

3.1.7.b <u>Numerical Example</u>:

Assuming a transition probability, the matrix P is:

$$P = \begin{vmatrix} 0 & 0.2 & 0.8 & 0 \\ 0.3 & 0 & 0 & 0.7 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Applying the technique used in the previous section:

$$W_e(s) = \frac{0.5 x 0.8e^s + 0.5 x 0.3e^s x 0.8e^s}{(1-.2e^s x 0.3e^s)}$$

$$P_e = W_e(s)\Big|_{s=0} = \frac{0.52}{0.94} = \frac{52}{94}$$

$$M_e(s) = \frac{W_e(s)}{W_e(s)\Big|_{s=0}}$$

The expected delay before the process is trapped in state three: $\tilde{V}\Big|_3$

$$\tilde{V}\Big|_3 = \left.\frac{dM_e(s)}{ds}\right|_{s=0}$$

$$= \frac{52}{94}\left(\frac{0.4e^{s}}{(1-0.06e^{2s})} + \frac{0.4\text{x}0.06\text{x}2e^{3s}}{(1-0.06e^{2s})^2}\right.$$

$$\left.+ \frac{0.12\text{x}2e^{2s}}{(1-0.06e^{2s})} + \frac{0.12\text{x}0.06\text{x}2\text{x}e^{4s}}{(1-0.06e^{2s})^2}\right)\Bigg|_{s=0} = 1.358$$

3.1.7.c Probabilities of Delay:

Considering the numerical example of section 3.1.7.b, a GERT 111 simulation program can be written to evaluate,

1. $P_3(n)$ - The probability that the process gets into trapping state three after transition.

2. $P_4(n)$ - The probability that the process gets into trapping state four after transitions.

3. $P_{3,4}(n)$ - The probability that the process gets in any of either trapping state three or four in n transitions.

The GERT 111 network representing the problem is shown in Fig. 9.3, where node 3 represents trapping state four and node 4 represents trapping state four.

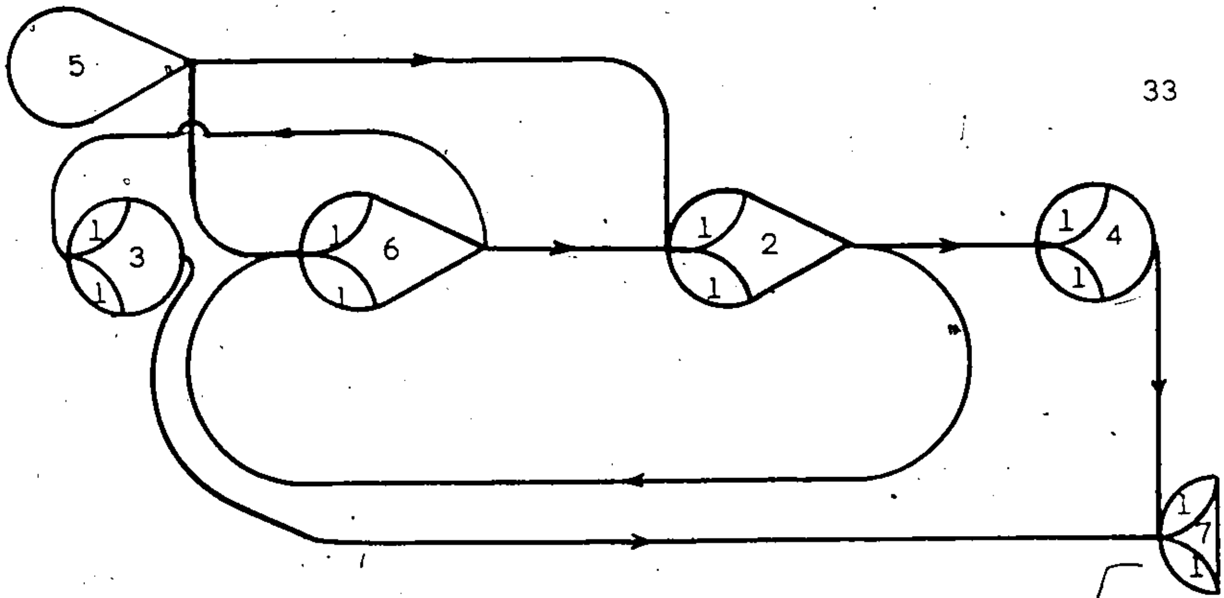

Fig. 9.3 GERTS 111 network representing the case of two trapping states.

The GERTS 111 program is presented in Fig. 6(A). The network is simulated 1000 times. The output, Fig. 7(A) shows that node 3 was realized 373 times after one transition, 127 times after two transitions, etc... These results may be interpreted as, after one transition the probability that the process will get in trapping state three is 373/1000 = 0.373, and the probability that the process will get in trapping state three after two transitions is 127/1000 = 0.127, etc...

Similarly the probability that the process will get in trapping state four in n transitions can be found. Likewise from the histogram statistics on node 7, the probability that process will get in either its trapping states after transitions, is evaluated.

The GERT 111 simulation results are summarized in Table 6.3. Values of $P_3(n)$, $P_4(n)$ and $P_{3,4}(n)$ obtained by mathematical evaluations are shown in Table 7.3.

n	0	1	2	3	4	5
$P_3(n)$	0	0.373	0.127	.026	.003	0
$P_4(n)$	0	0.371	0.067	0.026	0.005	0.001
$P_{3,4}(n)$	0	0.744	0.194	0.052	0.008	0.001

Table 6.3 GERT 111 simulation results.

n	0	1	2	3	4	5
$P_3(n)$	0	0.4	0.12	0.024	.0072	.00144
$P_4(n)$	0	0.35	0.07	0.021	.0042	.00126
$P_{3,4}(n)$	0	0.75	0.19	0.045	.0114	.0027

Table 7.3 Theoretical values of trapping probabilities.

The statistics collected on node 7 shows that the mean time it takes to realize node 7 is 1.3310, i.e., the expected total delay in the transient process regardless of in which state the process will be trapped is:

$$\bar{V} = 1.3310$$

Value of $\bar{V}$ obtained by mathematical evaluation is:

$$\bar{V} = 1.330$$

Statistics collected on node 3 shows that the probability of realizing node 3, i.e. the probability of the process to be trapped in state three is:

$$P_3 = 0.5290$$

In the same manner, from the statistics collected on node 4:

$$P_4 = 0.4710$$

Values of P_3 and P_4 obtained by mathematical evaluations are:

$$P_3 = 0.553 \quad \text{and} \quad P_4 = 0.447$$

To test the hypothesis that the evaluated values of $P_3(n)$, $P_4(n)$ and $P_{3,4}(n)$ are acceptable estimates for their expected values, the Chi-square test of hypothesis was applied in each case.

In testing for $P_3(n)$ the resulting Chi-square value was 6.12, which is well below the critical value of 9.23 associated with the 10% significance level for five degrees of freedom.

In testing for $P_4(n)$ the resulting Chi-square value was 2.73, which is well below the same critical value 9.236 as above.

In testing for $P_{3,4}(n)$ the resulting Chi-square value was 2.23, which is well below the critical value 9.23 as given above.

Thus the values of $P_n(n)$, $P_4(n)$ and $P_{3,4}(n)$ evaluated by GERT III were found to be acceptable estimates of their expected values.

CHAPTER IV

CASE STUDY OF THE WINDSOR STAR

4.1 Introduction:

The information was gathered by the Audit Bureau of Circulations, Chicago, Illinois, in its audit report, which is the basic measure of Circulation Values. The information was obtained through statements reported to the Audit Bureau by The Windsor Star, examined and substantiated by the Audit, in accordance with the Bureau's by-laws, rules and auditing standards, and included such tests of the accounting records and such other auditing procedures as were considered necessary in the circumstances.

Some of the questions on the statement are optional and need not be answered.

These reports are compiled for every newspaper in North America in a separate statement containing information for a 12-month period ending March 31st of each year.

The Windsor Star is classified as an evening general newspaper; it was established in Windsor in 1918 by W.F. Herman.

For reasons of clarification, it is necessary to define the terms used in the study.

4.2 Definitions:

(1) City Zone comprises the corporate limits of Windsor, plus the town of Tecumseh and the Village of St-Clair Beach, plus Sandwich West Township, as shown in Fig. 1.4.

(2) Retail Trading Zone is, with the exception of the City Zone: Essex County, Kent County, Townships of Raleigh, Romney, Tilbury East and Villages of Erieau, Erie Beach and Wheatly, as shown in Fig. 1.4.

(3) Adjusted Figures of country, census division or district totals are figures which have been arrived at by decreasing (or increasing) the listed country, census division or district total figures by the appartaining percentages calculated by measuring the gross distribution for one day only and is greater or less than the average paid for the period covered by each report (12 months). These adjusted county, census division or district totals will, therefore, approximate the average paid for the period covered by each report.

(4) Balance in county, census division or district is comprised of the distribution in towns receiving less than 25 copies which is not identified with the towns, townships or minor civil divisions listed.

(5) Motor route or mail circulation is identified by the town at which the motor route starts or at which it ends, or is the Post Office address of mail subscribers. In either

city and retail trading zones / WINDSOR, ONTARIO, CANADA

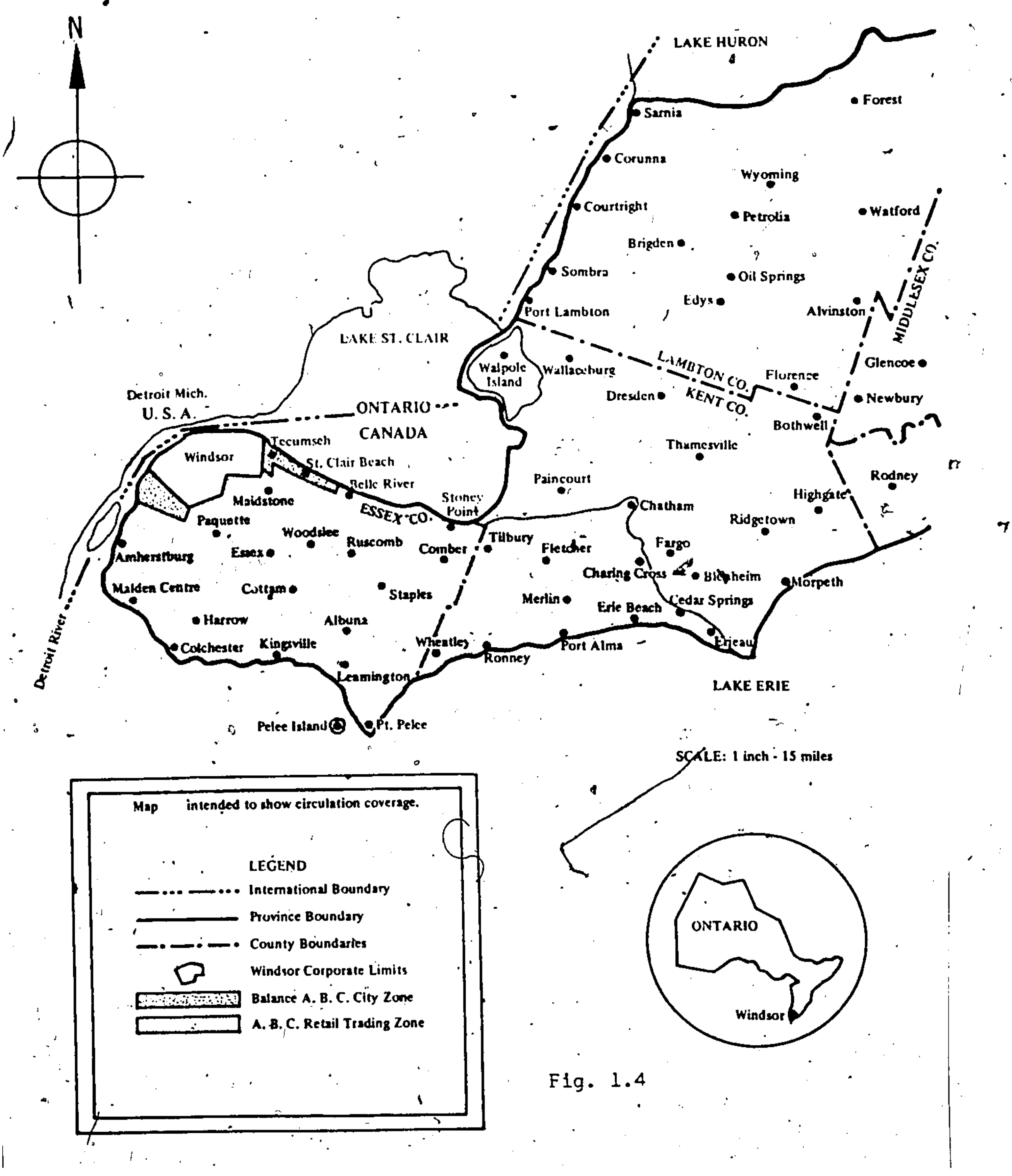

Fig. 1.4

event, it does not mean that all the copies are delivered in that town. Motor route circulation figures include copies dropped along the road to individual subscribers and mail circulation figures include copies served on Post Office Rural Routes emanating from the Post Office named.

(6) Net press run contains all produced copies, i.e., copies spoiled in distribution, free copies, unsold copies and production allowances.

(7) Sales release in the City Zone is immediate. In the Retail Trading Zone and all other trading zones, sales release is made on arrival at destination.

(8) Average Bulk Sales in all Zones represents copies sold to various individuals and to business concerns and to local air lines for free use of its passengers at 10¢ per copy, distribution being made by purchasers.

(9) Average Unpaid Distribution consists of arrears, service advertisers, employees, agencies, samples, etc...

(10) Special Reduced Prices: for each 1-year, 6-month, 3-month, 1-week subscription.

(11) Subscriber is, in essence, any person, group of persons, or business establishment that has the newspaper delivered on a regular basis by paperboy, by route truck or by mail.

(12) Cancellation is the termination of regular delivery, provided that any temporary interruption of regular

delivery to a person, such as for vacations or other temporary absence of the person from the address to which regular delivery is made, shall not be deemed a termination of regular delivery. Subscription shall be deemed to be a continuing subscription as long as regular delivery is maintained to any member of a household continuing to reside as a part thereof. If any member moves permanently from a household, and if regular delivery is maintained to any member of the household, the subscription is not to be deemed terminated. For the purposes of this definition, an interruption of regular delivery for an indefinite period, expected to exceed one year, shall not be deemed to be a temporary interruption of regular delivery.

4.3 Construction of the Markov Models:

The aggregate data obtained from the five consecutive A.B.C. reports issued between 1969 and 1973 were used to construct the Markov models, on the assumption that the behavior pattern of subscribers is reflected by stationary first order Markov process. Application of the Chi-square test of the hypothesis of a stationary chain [redacted] applied in each case to validate the above assumption. In each case, the number of degrees of freedom associated with the evaluated Chi-square is:

$$\gamma = r(r-1)(t-1)$$

where:

γ is the number of degrees of freedom

r is the number of states (equals 2 in each case)

t is the number of time intervals between the data gathering (equals 4 in each case)

Thus, the number of degrees of freedom in each case is identical and equals 6.

In the construction of the Markov models, any householder is considered to be in any one of two states: either in state 1 if being a subscriber, or in state 2 if being a non-subscriber.

Since the A.B.C. reports contain aggregate data for each year ending on March 31st, the constructed models are discrete in time and the transition time is one year. The data gathered from the A.B.C. reports were tabulated in the form of augmented matrices of proportions, subscribers versus the total number of householders in each case, except for the City Zone. The augmented matrix will be proportions of subscribers versus the population census of the City Zone, since each household subscribes in the average to more than one copy, as shown in Tables 1(C)a to 1(C)g.

The computer program in Reference [10] is employed to evaluate the transition probability matrices for the City Zone, Retail Trading Zone, the four southern counties of Ontario: Essex, Kent, Lambton and Middlesex, and for the total subscriptions of The Windsor Star in Southern Ontario. Each case is considered individually.

4.3.1a The City Zone:

The computer output for the evaluated matrix employing the unrestricted estimator and the Chi-square value in the case of the City Zone are shown in Fig.8(A)a. The estimated transition probability matrix is

$$\hat{P}_{C.Z.} = \begin{vmatrix} 0.625 & 0.375 \\ 0.134 & 0.866 \end{vmatrix}$$

where the probabilities are rounded to 3 decimal figures, the resulting Chi-square value is .000602, which is well below the critical value 10.64 associated with the 10% significance level for six degrees of freedom.

Thus the hypothesis that $\hat{P}_{C.Z.}$ is an estimate of a first order stationary transition probability matrix that prescribes subscribers' pattern to The Windsor Star in the City Zone is accepted.

4.3.1b GERT III Simulation Analysis for the City Zone Markov Model:

In this case, the estimated probability matrix does not contain a trapping state, therefore it can be represented by GERT III elements in the same manner as in Fig. 1.3 where node 3 represents a dweller in the City Zone, being a subscriber. Node 4 represents the state of being a non-subscriber.

The GERT III computer program is presented in Fig.8(A)b and the output is presented in Fig.8(A)c.

In the same manner as in Section 3.1.2a, values of $\Phi_{11}(K/n)$ are evaluated, i.e., the joint probability distribution of a dweller of the City Zone will be a subscriber k times in n years, which is the probability distribution to be sought (values of $\Phi_{11}(K/n)$ as shown in Table 2(c)) since state 2 (of being a non-subscriber) is not a trapping state. Hence subscribers may cancel their subscriptions, then re-subscribe in the future such that this process may go on for an indefinite period of time. In such case, the probability of being a subscriber or a non-subscriber will reach a steady state. The steady state probability vector was found to be

$$\left| 0.263 \quad 0.737 \right| .$$

In the same manner as shown in Section 3.1.3 - $\sum_{K=0}^{\infty}\Phi_{11}(K/n) = \Phi_{11}(n)$, the n-step transition probability from state 1 to state 2 values $\Phi_{11}(n)$ are shown at the bottom of Table 2(C).

4.3.2a <u>The Retail Trading Zone</u>:

The computer output for the evaluated matrix employing the unrestricted estimator and the Chi-square value in the case of the Retail Trading Zone is shown in Fig.9(A)a. The estimated transition probability matrix is

$$\hat{P}_{R.T.Z.} = \begin{vmatrix} 0.718 & 0.282 \\ 0.256 & 0.744 \end{vmatrix}$$

The resulting Chi-square value is 0.014054, which is well below the critical value 10.64 associated with the 10% significance level for six degrees of freedom.

Thus the hypothesis that $\hat{P}_{R.T.Z.}$ is an estimate of a first order stationary transition probability matrix that prescribes subscribers' pattern to The Windsor Star in the Retail Trading Zone is accepted.

4.3.2b <u>GERT III Simulation Analysis for the Retail Trading Zone Markov Model</u>:

The Markov model can be analyzed in an identical manner as for the case of the City Zone model.

The GERT III computer program is presented in Fig.9(A)b and the output is presented in Fig.9(A)c.

Values of $\Phi_{11}(K/n)$ are shown in Table 3(C). The steady state probability vector is $|0.475 \quad 0.525|$.

Values of $\Phi_{11}(n)$ are shown at the bottom of Table 3(C).

4.3.3a <u>Essex County</u>:

The computer output for the evaluated matrix employing the unrestricted estimator and the Chi-square values in the case of Essex County are shown in Fig.10(A)a. The estimated transition probability matrix is

$$\hat{P}_{E.C.} = \begin{vmatrix} 0.431 & 0.569 \\ 0.963 & 0.037 \end{vmatrix}$$

The resulting Chi-square value is 0.009513, which is well below the critical value 10.64 associated with the 10% significance level for six degrees of freedom.

Thus the hypothesis that $\hat{P}_{E.C.}$ is an estimate of a first order stationary transition probability matrix that prescribes subscribers' pattern to The Windsor Star in Essex County is accepted.

4.3.3b GERT III Simulation Analysis for the Essex County Markov Model:

Also in this case the Markov model can be analyzed in an identical manner as for the case of the City Zone model.

The GERT III computer program is presented in Fig.10(A)b and the output is presented in Fig.10(A)c.

Values of $\Phi_{11}(K/n)$ are shown in Table 4(C). The steady state probability vector is $\begin{vmatrix} 0.629 & 0.371 \end{vmatrix}$.

Values of $\Phi_{11}(n)$ are shown at the bottom of Table 4(C).

4.3.4a Kent County:

The computer output for the evaluated matrix employing the unrestricted estimator and the Chi-square value in the case of Kent County are shown in Fig.11(A)a. The estimated transition probability matrix is

$$\hat{P}_{K.C.} = \begin{vmatrix} 0.704 & 0.296 \\ 0.081 & 0.919 \end{vmatrix}$$

The resulting Chi-square value is 0.000357, which is well below the critical value 10.64 associated with the 10% significance level for six degrees of freedom.

Thus the hypothesis that $\hat{P}_{K.C.}$ is an estimate of a first order stationary transition probability matrix that prescribes subscribers' pattern to The Windsor Star in Kent County is accepted.

4.3.4b <u>GERT III Simulation Analysis for the Kent County Markov Model</u>:

Also in this case the Markov model can be analyzed in an identical manner as for the case of the City Zone model.

The GERT III computer program is presented in Fig.11(A)b and the output is presented in Fig.11(A)c.

Values of $\Phi_{11}(K/n)$ are shown in Table 5(C). The steady state probability vector is $|0.215 \quad 0.785|$.

Values of $\Phi_{11}(n)$ are shown at the bottom of Table 5(C).

4.3.5a <u>Lambton County</u>:

The computer output for the evaluated matrix employing the generalyzed least square estimates and the Chi-square value in the case of Lambton County are shown in Fig.12(A)a. The estimated transition probability matrix is

$$\hat{P}_{L.C.} = \begin{vmatrix} 0.839 & 0.161 \\ 0.0 & 1.0 \end{vmatrix}$$

The resulting Chi-square value is 0.001996, which is well below the critical value 10.64 associated with the 10% significance level for six degrees of freedom.

Thus the hypothesis that $\hat{P}_{L.C.}$ is an estimate of a first order stationary transition probability matrix that prescribes subscribers' pattern to <u>The Windsor Star</u> in Lambton County is accepted.

4.3.5b <u>GERT III Simulation Analysis for the Lambton County Markov Model</u>:

-In this case the estimated probability matrix contains a trapping state, which is state two. Therefore, if a householder in Lambton County cancels his subscription, there is a negligible probability to resubscribe again any time in the future.

The Markov model can be represented by GERT III elements as shown in Fig.1(B), where node three represents state one, i.e., a householder being a subscriber. Node 5 represents trapping state two.

With the same understanding of section 3.1.6e, P(n) is the probability distribution to be sought, i.e., the probability that a subscriber gets into the trapping state of non-subscription in n transitions (years).

The GERT III simulation program is shown in Table 12(A)b. Values of P(n) are shown in Table 6(C).

Also in the same manner as in section 3.1.6e, the value of $\bar{V}$ (the expected number of transitions (years) before a subscriber gets into the trapping state of non-subscription) can be evaluated from the computer output shown in Fig.12(A)c where

$$\bar{V}_{L.C.} \approx 6 \text{ years}$$

4.3.6a Middlesex County:

The computer output for the evaluated matrix employing the unrestricted estimator and the Chi-square value in the case of Middlesex County are shown in Fig.13(A)a. The estimated transition probability matrix is

$$\hat{P}_{M.C.} = \begin{vmatrix} 0.533 & 0.467 \\ 0.0 & 1.0 \end{vmatrix}$$

The resulting Chi-square value is 0.000068, which is well below the critical value 10.64 associated with the 10% significance level for six degrees of freedom.

Thus the hypothesis that $P_{M.C.}$ is an estimate of a first order stationary transition probability matrix that prescribes subscribers' pattern to The Windsor Star in Middlesex County is accepted.

4.3.6b GERT III Simulation Analysis for the Middlesex County Markov Model:

This case is identical to the Lambton County case, where state two is a trapping state. Therefore, if a householder in Middlesex County cancels his subscription, there is little chance that he will resubscribe.

The Markov model can also be represented by GERT III elements, as shown in Fig.1(B). The GERT III simulation program is shown in Fig.13(A)b; the output is shown in Fig.13(A)c. Values of P(n) are shown in Table 7(C). In this case,

$$\bar{V}_{M.C.} \approx 2 \text{ years}$$

4.3.7a Southern Ontario (total):

The computer output for the evaluated matrix employing the unrestricted estimator and the Chi-square values in the case of Southern Ontario are shown in Fig.14(A)a. The estimated transition probability matrix is

$$P_{S.O.} = \begin{vmatrix} 0.699 & 0.301 \\ 0.128 & 0.872 \end{vmatrix}$$

The resulting Chi-square is 0.011257, which is well below the critical value 10.64 associated with the 10% significance level for six degrees of freedom.

Thus the hypothesis that $P_{S.O.}$ is an estimate of a first order stationary transition probability matrix that prescribes subscribers' pattern to The Windsor Star in Southern Ontario is accepted.

4.3.7b GERT III Simulation Analysis for Southern Ontario Markov Model:

Also in this case the Markov model can be analyzed in an identical manner as for the case of the City Zone model.

The GERT III computer program is presented in Fig.14(A)b and the output is presented in Fig.14(A)c. Values of $\phi_{11}(K/n)$ are shown in Table 8(C). The steady state probability vector is $|0.298 \quad 0.702|$.

Values of $\phi_{11}(n)$ are shown at the bottom of Table 8(c).

CHAPTER V

CASE STUDY OF THE DETROIT EVENING NEWS

5.1 Introduction:

As in the case of The Windsor Star, the prime task is to evaluate the probability transition matrix of subscribers. Fortunately, the probability transition matrix was evaluated by the management of The Detroit Evening News because this information was needed to settle a tax case; the information gathered is used in this case study.

The case of The Detroit Evening News differs from that of The Windsor Star in that insurance policies are offered with subscription to the paper.

5.2 Definition:

Cancellation is the termination of regular delivery; provided that any temporary interruption of regular delivery to a person, such as for vacations or other temporary absence of the person from the address to which regular delivery is made, shall not be deemed a termination of regular delivery.

Subscription shall be deemed to be a continuing subscription as long as regular delivery is maintained to any member of a household continuing to reside as a part thereof. If any member moves permanently from a household, and if regular delivery is maintained to any member of the household, the subscription is not to be deemed terminated. For purposes of this definition, an interruption of regular delivery for an indefinite period expected to exceed 6 months, shall not be deemed to be a temporary interruption of regular delivery.

5.3 Description of the Probability Transition Matrix:

Design of the Probability Sample:

The sample was selected by random numbers from the Audit Bureau of Circulations subscribers list for one year. Instructions to The Detroit Evening News with respect to the selection of the sample required that they number the pages of the A.B.C. list 1, 2, onward so that every page had a serial number. Within each zone of 100 consecutive pages, random numbers drew one page into each of the 5 sub-samples. For each page so selected, random numbers drew 2 of the 35 lines into the sample.

Different Groups of Subscribers:

It was decided to group subscribers on two main criteria:

i) According to whether the subscription was relatively recent. In this connection, subscriptions were being classified as:

a) less than 1 year old

b) from 1 to less than 2 years old

c) 2 or more years old.

This classification was accomplished by tracing names of subscribers that fell into the sample to A.B.C. lists of subscribers for past years.

ii) According to whether the subscriber carried News-sponsored Accident Insurance. Only subscribers to The Detroit Evening News are permitted to carry such a policy. Hence, subscribers that carry insurance may be more reluctant than others to give up their subscription, lest they lose their insurance as well.

Thus, in forming the model, a subscriber to The Detroit Evening News will be in one of 7 possible states in a Markov process, which are as follows:

1 - Subscriber, uninsured, subscription less than 1 year old.

2 - Subscriber, uninsured, subscription from 1 to less than 2 years.

3 - Subscriber, uninsured, subscription 2 or more years old.

4 - Subscriber, insured, subscription less than 1 year old.

5 - Subscriber, insured, subscription from 1 to less than 2 years.

6 - Subscriber, insured, subscription 2 or more years old.

7 - Subscription cancelled.

The nature of the model is that each subscriber to the News is in one of these 7 possible states at a given time. That is, the subscriber may be insured, or he may not be insured, and in either case he may have subscribed for less than 1 year, for less than 2 years, or for 2 years or more. Furthermore, a subscriber may, from one year to the next, shift from one state to any one of 2 or 3 other possible states. For example, a subscriber in state 3 may shift, by the next year, to state 6 or to 7, or he may remain in state 3. Other transitions, by their nature, are not possible.

The transition probability matrix is,

	1	2	3	4	5	6	7
1	0.0	0.740	0.0	0.0	0.015	0.0	0.245
2	0.0	0.0	0.84	0.0	0.0	0.014	0.145
3	0.0	0.0	0.928	0.0	0.0	0.013	0.059
4	0.0	0.0	0.0	0.0	0.857	0.0	0.143
5	0.0	0.0	0.0	0.0	0.0	0.833	0.167
6	0.0	0.0	0.043	0.0	0.0	0.920	0.037
7	0.0	0.0	0.0	0.0	0.0	0.0	1.0

where state seven is a trapping state. Once a subscriber moves to state 7 (cancellation), no further transitions are possible.

5.4 GERT III Simulation Analysis for The Detroit Evening News:

Seven different situations may be considered in this case, as follows:

(1) A subscriber is in state 1

(2) A subscriber is in state 2, etc... to number 6

(7) A random subscriber who could be in any of the above states.

In each of the above cases, values of P(n), i.e., the probability that a subscriber who is either in one of the first six states, or a random subscriber, gets into the trapping state 7 of cancellation in n transitions (Years) is the probability distribution to be sought. The first six situations are discussed in this section, while the situation of a random subscriber is discussed in the following section.

The first 6 of the above cases are represented by GERT III elements as shown in Fig.2(B) through 7(B) respectively, where nodes 2,3,4,5,6,7,8, represent states 1,2,3,4,5,6,7.

The computer programs are presented in Fig.15(A)b through 20(A)b respectively. The GERT III simulation outputs are presented in Fig.15(A)c through 20(A)c respectively, from which values of P(n) are evaluated, in the same manner as discussed in section 3.1.6e. Values of P(n) for the first 6 cases are shown in Tables 9(C) through 14(C), respectively.

Values of $\bar{V}_i$ (the expected number of transitions (years) before the subscriber gets into the trapping state 7 of non-subscriber) are evaluated from the GERT III simulation output in Fig.15(A)c through 20(A)c. Values of $\bar{V}$ are:

$$\bar{V}_1 = 12.3810$$
$$\bar{V}_2 = 15.0490$$
$$\bar{V}_3 = 16.6720$$
$$\bar{V}_4 = 16.6530$$
$$\bar{V}_5 = 18.2670$$
$$\bar{V}_6 = 21.0530$$

where

$\bar{V}_1$ pertains to the first case

$\bar{V}_2$ pertains to the second case, etc...

5.5 The Subscription Life of a Random Subscriber:

It is interesting to evaluate the probability distribution P(n) of a random subscriber. A random sample is drawn from the subscribers' list - the sample is examined to determine the proportions of insured and uninsured subscribers. In both of these groups, a certain proportion of those had subscribed for less than 1 year, for less than 2 years, and for two years or more. A 7-component probability vector is introduced to describe the composition of this reference group. The components of this vector show the proportion of subscribers observed in the states 1, 2, 3, 4, 5, 6, 7. The vector used here is:

$$\left| .2083 \quad .1274 \quad .5070 \quad .0032 \quad .0081 \quad .1460 \quad .0000 \right|.$$

Each component of the vector is the probability that a subscriber, selected at random, will be in a given state.

This case can be analyzed in a similar manner as discussed in the previous section. The model represented by GERT III elements is shown in Fig.8(B); the GERT III simulation program and output are shown in Fig.21(A)b and 21(A)c respectively.

Values of P(n) are evaluated in the same manner as the preceding section, as shown in Table 15(C). In this case,

$$\bar{V}_{R.S.} = 16.0480$$

CHAPTER VI

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

6.1 Conclusions:

It has been demonstrated that GERT III is well suited for analyzing with east Markov models possessing a relatively large number of states which are far beyond the scope of other conventional techniques of analysis.

In analyzing the life of subscriptions to The Windsor Star, it was found that the transition probability matrices for the City Zone, the Retail Trading Zone, and the Counties of Essex and Kent, as shown in Figs.8(A)a, 9(A)a, 10(A)a and 11(A)a of the computer outputs, have no trapping states. As a consequence, the typical householder alternated between subscribing and not subscribing. Practically speaking, then, a subscription is indeterminate. This is undoubtedly due to the monopolistic position The Windsor Star enjoys in Essex and Kent Counties.

On the other hand, in Lambton and Middlesex Counties, the subscription life, as shown from the GERT III computer outputs Figs.12(A)c and 13(A)c is approximately 6 years and 2 years respectively. Apparently this is due to the competition with other papers, e.g., The London Free Press and The Sarnia Observer.

The study showed that The Detroit Evening News subscribers who were insured kept their subscriptions for a longer time than subscribers who were not. The subscription life for The Detroit Evening News are:

State Subscriber is in	Subscription life (in yrs.)
Uninsured subscriber, less than 1 year old	12
Uninsured subscriber, from 1 to less than 2 years	15
Uninsured subscriber, 2 or more years old	17
Insured subscriber, less than 1 year	17
Insured subscriber, from 1 to less than 2 years	18
Insured subscriber, 2 or more years old	21
Random subscriber	16

It was found that The Windsor Star subscriptions and The Detroit Evening News are in no way comparable.

In the newspaper industry, perhaps as in other industries, monopoly has a most important effect on the life of subscriptions.

6.2 Suggestions for Further Research:

The application of GERT and GERT III to semi-Markov processes with both continuous and discrete times, would no doubt prove to be of significance. It is also recommended that the application of GERT III to the analysis of Markov processes of orders higher than the first order be studied. It is suggested that GERT III can be manipulated to analyze Markov processes where rewards and sequential decisions are involved.

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APPENDIX (A)

COMPUTER PROGRAMS AND OUTPUTS

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	0.0	0.0	0.0	0.0
2	1.0000	0.0	0.0	0.0
3	1.0000	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0
5	1.0000	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0
8	1.0000	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	1	0	0	1.0000
3	5	3	1	0	0	0.8000
3	4	2	1	0	0	0.2000
4	6	8	1	0	0	0.7000
4	7	5	1	1	0	0.3000
5	3	4	1	0	0	1.0000
5	8	10	1	0	0	1.0000
6	4	9	1	0	0	1.0000
7	8	6	1	0	0	1.0000
7	3	7	1	0	0	1.0000
8	9	11	1	0	0	1.0000

Fig. 1(A)

GERT SIMULATION PROJECT -1 BY O.EL GOTHAMY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT		MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000		1.6950	1.7587	1000.	1.0000	14.0000	A
9		1	0.2080	0.4061	1000.	0.0	1.0000	
4	0.2080		3.0705	2.2015	695.	1.0000	13.0000	A
4		1	0.0	0.0	695.	0.0	0.0	
3	1.0000		0.8475	1.5048	2000.	0.0	14.0000	A
3		1	0.1040	0.3053	2000.	0.0	1.0000	

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	792	60	37	35	23	16	12	6	11	2
			4	1	0	1	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	
4	1.00	1.00	0	208	148	111	76	53	37	25	19	8	6
			2	1	1	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	
3	1.00	1.00	1000	792	60	37	35	23	16	12	6	11	2
			4	1	0	1	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	

Fig. 2(A)

GERT SIMULATION PROJECT -1 BY O.EL GOTHAMY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	3.3400	2.5276	1000.	2.0000	20.0000	A
9	1	0.3960	0.5563	1000.	0.0	2.0000	
4	0.3610	4.0769	2.8042	1340.	1.0000	18.0000	A
4	1	0.0761	0.2653	1340.	0.0	1.0000	
3	1.0000	1.6537	2.2536	3000.	0.0	20.0000	A
3	1	0.1923	0.4228	3000.	0.0	2.0000	

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	2.00	1.00	0	639	109	55	46	41	25	20	27	12	7
			6	5	3	1	1	2	0	0	1	0	0
			0	0	0	0	0	0	0	0	0	0	
4	1.00	1.00	0	181	309	216	177	126	100	72	57	30	24
			16	12	7	5	4	2	1	1	0	0	0
			0	0	0	0	0	0	0	0	0	0	
3	2.00	1.00	1819	691	145	75	71	51	38	28	35	14	10
			7	6	3	1	2	2	0	1	1	0	0
			0	0	0	0	0	0	0	0	0	0	

Fig. 2(A)a

GERT SIMULATION PROJECT -1 BY O.EL GOTHAMY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	5.1390	3.2824	1000.	3.0000	24.0000	A
9	1	0.6230	0.6952	1000.	0.0	3.0000	
4	0.5100	5.0337	3.5937	2139.	1.0000	23.0000	A
4	1	0.2034	0.4382	2139.	0.0	2.0000	
3	1.0000	2.5972	3.0430	4000.	0.0	24.0000	A
3	1	0.3167	0.5463	4000.	0.0	3.0000	

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	3.00	1.00	0	490	114	104	71	42	41	34	27	21	14
			12	9	2	2	4	5	3	1	1	1	0
			2	0	0	0	0	0	0	0	0	0	
4	1.00	1.00	0	217	331	362	276	225	172	140	99	72	65
			45	33	25	18	17	13	11	6	4	3	2
			2	1	0	0	0	0	0	0	0	0	
3	3.00	1.00	2452	638	234	171	120	81	80	66	39	32	23
			17	12	4	6	6	7	4	2	2	1	1
			2	0	0	0	0	0	0	0	0	0	

Fig. 2(A)b

GERT SIMULATION PROJECT -1 BY H.EL GOTHAMY
DATE 4/ 29/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT		MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000		6.4200	3.3792	1000.	4.0000	29.0000	A
9		1	0.7670	0.7908	1000.	0.0	4.0000	
4	0.5750		5.5467	3.8225	2420.	1.0000	28.0000	A
4		1	0.2880	0.5388	2420.	0.0	3.0000	
3	1.0000		3.2198	3.2718	5000.	0.0	29.0000	A
3		1	0.3858	0.6212	5000.	0.0	4.0000	

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	4.00	1.00	0	425	141	98	68	59	54	32	37	28	16
			8	11	2	6	5	2	1	3	2	1	0
			1	0	0	0	1	0	0	0	0	0	
4	1.00	1.00	0	202	286	332	347	294	225	175	134	94	85
			62	45	31	26	20	19	10	6	7	6	4
			2	2	2	1	1	1	1	0	0	0	
3	4.00	1.00	3180	653	281	209	161	134	115	70	61	41	27
			16	14	4	11	9	3	2	3	3	1	0
			1	0	0	0	1	0	0	0	0	0	

Fig. 2(A)c

GERT SIMULATION PROJECT -1 BY O.EL GOTHAMY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT		MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000		8.3300	4.0907	1000.	5.0000	38.0000	A
9		1	1.0210	0.8873	1000.	0.0	4.0000	
4	0.6850		16.6766	4.6278	3330.	1.0000	37.0000	A
4		1	0.4120	0.6303	3330.	0.0	3.0000	
3	1.0000		4.1642	4.0403	6000.	0.0	38.0000	A
3		1	0.5125	0.7134	6000.	0.0	4.0000	

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	5.00	1.00	0	315	147	91	84	65	72	51	37	38	22
			17	14	13	3	9	5	3	2	1	4	2
			1	0	1	1	1	0	0	0	0	1	
4	1.00	1.00	0	200	328	354	359	373	325	272	230	181	160
			120	85	70	61	48	36	25	23	16	10	10
			9	8	4	4	4	4	3	1	1	6	
3	5.00	1.00	3759	627	360	266	217	182	138	106	90	68	39
			30	25	22	11	15	12	7	5	4	7	3
			1	0	1	2	1	0	0	1	0	1	

Fig. 2(A)d

GERT SIMULATION PROJECT -1 BY O.EL GETHAMY
DATE 4/ 30/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT		MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000		9.8590	4.2586	1000.	6.0000	37.0000	A
9		1	1.1910	0.9610	1000.	0.0	4.0000	
4	0.7450		7.1925	4.6668	3859.	1.0000	36.0000	A
4		1	0.4680	0.6813	3859.	0.0	3.0000	
3	1.0000		4.9760	4.5031	7000.	0.0	37.0000	A
3		1	0.6049	0.7850	7000.	0.0	4.0000	

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	6.00	1.00	0	255	145	93	81	80	76	57	38	35	31
			33	13	20	9	3	5	5	7	0	2	1
			2	0	2	0	1	0	0	0	0	1	
4	1.00	1.00	0	196	326	373	368	371	388	316	272	241	207
			175	137	117	88	70	47	39	33	20	18	15
			8	8	[illegible]	4	4	3	2	1	1	5	
3	6.00	1.00	4366	612	429	328	266	219	171	133	96	87	70
			62	37	30	23	16	11	13	8	3	5	3
			3	2	3	1	1	0	0	1	0	1	

Fig. 2(A)e

GERT SIMULATION PROJECT -1 BY O.EL GOTHAMY
DATE 4/ 20/ 1974.

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT		MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000		11.8660	5.0197	1000.	7.0000	41.0000	A
9		1	1.4320	1.0509	1000.	0.0	5.0000	
4	0.8000		8.5023	5.5183	4866.	1.0000	39.0000	A
4		1	0.5843	0.7673	4866.	0.0	4.0000	
3	1.0000		5.9435	5.3127	8000.	0.0	41.0000	A
3		1	0.7134	0.3787	8000.	0.0	5.0000	

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	7.00	1.00	0	200	110	93	93	88	76	49	50	45	29
			39	29	24	11	11	11	11	7	2	4	4
			4	1	3	0	1	1	1	1	0	2	
4	1.00	1.00	0	293	317	356	363	405	401	411	373	317	278
			259	207	189	153	122	101	76	67	52	46	37
			33	26	19	14	9	6	7	5	4	19	
3	7.00	1.00	4955	589	427	373	319	254	209	151	138	119	95
			91	61	47	29	27	20	16	12	10	13	12
			7	5	5	1	3	3	1	2	1	5	

Fig. 2(A)f

GERT SIMULATION PROJECT -1 BY D.EL GOTHAMY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT		MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000		13.3930	5.0012	1000.	8.0000	41.0000	A
9		1	1.6410	1.1548	1000.	0.0	6.0000	
4	0.8370		9.0534	5.4949	5393.	1.0000	37.0000	A
4		1	0.7089	0.8755	5393.	0.0	5.0000	
3	1.0000		6.6723	5.6349	9000.	0.0	41.0000	A
3		1	0.8133	0.9708	9000.	0.0	6.0000	

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	8.00	1.00	0	163	106	92	79	79	72	60	51	62	53
			39	27	26	17	14	18	10	5	5	6	5
			2	5	0	0	1	1	0	0	1	1	
4	1.00	1.00	0	205	309	328	377	373	400	388	395	404	345
			301	280	253	201	173	142	105	87	75	53	42
			34	29	25	19	15	9	7	7	1	11	
3	8.00	1.00	5620	605	433	386	338	280	228	208	176	156	131
			96	69	64	49	40	31	17	13	12	13	9
			4	8	2	2	3	2	0	0	1	4	

Fig. 2(A)g

GERT SIMULATION PROJECT -1 BY O.EL GOTHAMY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT		MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000		15.1330	5.4625	1000.	9.0000	43.0000	A
9		1	1.8460	1.2214	1000.	0.0	6.0000	
4	0.8750		9.8780	6.1036	6133.	1.0000	41.0000	A
4		1	0.7784	0.9450	6133.	0.0	5.0000	
3	1.0000		7.6393	6.3234	10000.	0.0	43.0000	A
3		1	0.9149	1.0442	10000.	0.0	6.0000	

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	9.00	1.00	0	125	87	85	83	85	87	61	55	60	50
			44	35	32	13	17	14	10	11	6	9	8
			7	3	3	0	2	3	2	0	0	3	
4	1.00	1.00	0	181	295	372	384	390	420	418	424	426	395
			365	326	274	235	212	182	144	130	104	95	66
			55	47	46	36	24	22	19	-10	8	38	
3	9.00	1.00	6116	574	480	423	377	346	300	236	205	188	142
			118	93	77	56	51	35	21	33	24	21	21
			15	9	6	4	5	4	3	1	1	5	

Fig. 2(A)h

GERT SIMULATION PROJECT -1 BY J.EL GOTHAMY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT		MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000		16.6630	5.8178	1000.	10.0000	48.0000	A
9		1	2.0110	1.2588	1000.	0.0	7.0000	
4	0.9000		10.8481	6.8269	6663.	1.0000	47.0000	A
4		1	0.9221	1.0570	6663.	0.0	6.0000	
3	1.0000		8.3441	6.7190	11000.	0.0	48.0000	A
3		1	1.0130	1.1018	11000.	0.0	7.0000	

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	10.00	1.00	0	100	96	79	78	78	72	65	61	66	54
			41	35	33	31	20	16	15	10	10	5	6
			4	4	2	3	3	3	4	0	1	6	
4	1.00	1.00	0	199	292	338	372	406	408	391	383	406	407
			391	340	319	283	273	249	214	163	135	118	96
			76	61	58	47	39	35	27	22	18	97	
3	10.00	1.00	6805	593	509	464	406	364	296	248	218	208	179
			133	114	99	81	53	44	36	25	23	18	17
			12	10	7	9	7	8	5	0	4	14	

Fig. 2(A)1

GERT SIMULATICN PROJECT 12 BY O.EL GOTHAMY
DATE: 4/ 20/ 1974

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 7
NUMBER OF SOURCE NODES IS 1
NUMBER OF SINK NODES IS 1
NUMBER OF NODES TO REALIZE THE NETWORK IS 1
STATISTICS COLLECTED CN 3 NODES.
NUMBER OF PARAMETER SETS IS 7
INITIAL RANDOM NUMBER IS 7777777 0.0

NOCE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	C	9999	D		
3	1	1	P		A
4	1	1	P		A
5	1	1	D		
6	1	1	D		
7	1	9999	D		A

SOURCE NODE NUMBERS
2

SINK NODE NUMBERS
7

STATISTICS COLLECTED ALSO CN NODES
4 3

Fig. 3(A)

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0
4	1.0000	0.0	0.0	0.0
5	1.0000	0.0	0.0	0.0
6	1.0000	0.0	0.0	0.0
7	1.0000	0.0	0.0	0.0

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	1	0	0	1.0000
3	5	4	1	0	0	0.8000
3	4	5	1	0	0	0.2000
4	6	6	1	0	0	0.7000
4	7	7	1	0	0	0.3000
5	3	2	1	0	0	1.0000
6	4	3	1	0	0	1.0000

Fig. 4(A)

GERT SIMULATION PROJECT 12 BY O.EL GOTHAMY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
7	1.0000	8.4080	5.4354	1000.	2.0000	44.0000	A
4	1.0000	7.6509	5.6856	3280.	1.0000	43.0000	A
3	1.0000	4.0571	4.5490	5128.	0.0	35.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
7	1.CC	1.00	0	0	61	79	111	106	91	91	76	54	57
			52	49	27	23	16	24	10	17	13	12	7
			4	3	2	1	2	3	2	2	0	5	
4	1.00	1.00	0	180	282	333	325	304	269	242	219	197	167
			138	102	86	75	67	51	52	38	31	20	15
			13	10	8	11	9	6	4	2	2	22	
3	1.00	1.00	1000	820	657	527	424	339	283	219	166	134	107
			84	71	60	48	40	32	21	18	12	11	9
			7	7	7	3	3	3	3	3	3	7	

Fig. 5(A)

GERT SIMULATION PROJECT -1 BY D.EL GOTHAMY
DATE 4/ 20/ 1974

NETWORK DESCRIPTION

NODE CHARACTERISTICS.

HIGHEST NODE NUMBER IS 7
NUMBER OF SOURCE NODES IS 1
NUMBER OF SINK NODES IS 1
NUMBER OF NODES TO REALIZE THE NETWORK IS 1
STATISTICS COLLECTED ON 3 NODES
NUMBER OF PARAMETER SETS IS 8
INITIAL RANDOM NUMBER IS 1783297 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	1	1	P		A
3	1	1	D		A
4	1	1	D		
5	1	1	P		
6	1	1	P		A
7	1	-1	D		

SOURCE NODE NUMBERS
5

SINK NODE NUMBERS
7

STATISTICS COLLECTED ALSO ON NODES
4 3

Fig. 6(A)

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0
5	1.0000	0.0	0.0	0.0
6	1.0000	0.0	0.0	0.0
7	1.0000	0.0	0.0	0.0
8	1.0000	0.0	0.0	0.0

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	4	5	1	0	0	0.7000
2	6	7	1	0	0	0.3000
3	7	4	1	0	0	1.0000
4	7	3	1	0	0	1.0000
5	2	1	1	0	0	0.5000
5	6	2	1	0	0	0.5000
5	3	8	1	0	0	0.8000
6	2	6	1	0	0	0.2000

Fig. 6(A)

GERT SIMULATION PROJECT -1 BY O.EL GOTHAMY
DATE 4/ 30/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
7	1.0000	1.3310	0.6371	1000.	1.0000	6.0000	A
4	0.4710	1.3036	0.6741	471.	1.0000	6.0000	A
3	0.5290	1.3554	0.6017	529.	1.0000	4.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
7	1.00	1.00	0	744	194	52	8	1	1	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	
4	1.00	1.00	0	371	67	26	5	1	1	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	
3	1.00	1.00	0	373	127	26	3	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	

Fig. 7(A)

TRANSITION MATRIX OF THE CITY ZONE

HE OBSERVED UNITS AND SAMPLE SIZES

	1	2	3
1	0.28280C0E 00	0.7172000E 00	0.1000000[illegible] 01
2	0.27960C0E 00	0.7204000E 00	0.1000000E 01
3	0.2650000E 00	0.7350000E 00	0.1000000E 01
4	0.26C3000E 00	0.73970[illegible]E C0	0.1000000E 01
5	0.26[illegible]1999E C0	0.7338000E 00	0.1000000E 01

S YOU DESIRED,COLUMN, 2IS DROPPED IN FORMING THE THE SYSTEM.
EAC THE FOLLOWING MATRICES IN THE SEQUENCE 1 2
THE WEIGHT MATRIX IS DERIVED FROM THE MLE WHERE Z=1 FORDIAGONALS AND Z=0 OTHERWISE.

INRESTRICTED ESTIMATOR OF THE TRANSITION MATRIX

0.62209308	0.37790692
0.13534755	0.86465245

Fig. 8(A)a

HE PRICICTED PROPORTIONS

	1	2
2	0.2731020E 00	0.7268978E 00
3	0.2715346E 00	0.7284652E 00
4.	0.2643830E 00	0.7356169E 00
5	0.2620807E 00	0.7379191E 00
6	0.2649708E 00	0.7350290E 00

UM OF SQUARED ERROR	MEAN SQUARED ERROR	CHI SQUAREVALUE	MODIFIED CHI SQUARE
0.000237126	0.000029641	0.000602024	0.000602302

HE UNRESTRICTED ESTIMATOR IS PERFECT.

RECURSIVE DIFFERENCE= 0.006183
RECURSIVE OUTPUT K= 2

UNRESTRICTED ESTIMATOR OF THE TRANSITION MATRIX

0.62477696	0.37522304
0.13444048	0.86555952

Fig. 8(A)a

GERT SIMULATION PROJECT 30 BY CITY ZONE
DATE 4/ 29/ 1974

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 9
NUMBER OF SOURCE NODES IS 1
NUMBER OF SINK NODES IS 1
NUMBER OF NODES TO REALIZE THE NETWORK IS 1
STATISTICS COLLECTED ON 1 NODES
NUMBER OF PARAMETER SETS IS 11
INITIAL RANDOM NUMBER IS 7777777 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	0	9999	D		
3	1	1	P		
4	1	1	P		
5	1	1	D		
6	1	1	D		
7	1	1	D		
8	1	9999	D		
9	1	9999	D		

SOURCE NODE NUMBERS
2

SINK NODE NUMBERS
9

Fig. 8(A)b

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0
8	1.0000	0.0	0.0	0.0
9	1.0000	0.0	0.0	0.0
10	1.0000	0.0	0.0	0.0
11	1.0000	0.0	0.0	0.0

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	1	0	0	1.0000
3	5	8	1	0	0	0.6250
3	4	9	1	0	0	0.3750
4	6	10	1	0	0	0.8660
4	7	11	1	0	0	0.1340
5	3	2	1	0	0	1.0000
5	8	3	1	0	0	1.0000
6	4	4	1	0	0	1.0000
7	8	5	1	0	0	1.0000
7	3	7	1	0	0	1.0000
8	9	6	1	0	0	1.0000

Fig. 8(A)b

GERT SIMULATION PROJECT 80 BY CITY ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	3.9700	5.7252	1000.	1.0000	39.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	9	612	51	41	53	27	25	25	17	20	16
			13	14	11	13	3	5	10	5	8	0	6
			3	3	3	2	2	2	1	1	2	6	

Fig. 8(A)c

GERT SIMULATION PROJECT 31 BY CITY ZONE
DATE 04/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	7.8000	7.8790	1000.	2.0000	50.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	372	80	47	50	51	47	17	36	28
			28	31	28	22	19	22	13	14	6	15	3
			6	9	5	10	4	3	3	3	3	25	

Fig. (A)c

GERT SIMULATION PROJECT 32 BY CITY ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	11.1410	9.3641	1000.	3.0000	63.0000	

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9.	1.00	1.00	0	0	0	239	53	61	67	60	41	45	41
			31	25	42	38	22	31	16	7	18	13	15
			12	14	10	12	9	6	4	9	4	55	

Fig. 8(A)c

GERT SIMULATION PROJECT 33-BY CITY ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	NO. OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	14.7700	10.4970	1000.	4.0000	59.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	159	46	43	50	46	39	54
			55	35	34	35	30	30	31	33	24	17	17
			01	14	10	15	18	23	14	9	12	96	

Fig. 8(A)c

GERT SIMULATION PROJECT 34 BY CITY ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	18.9190	12.1911	1000.	5.0000	82.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	98	36	36	33	37	42
			46	41	32	23	39	44	44	32	30	29	21
			23	29	16	33	18	17	18	16	11	156	

Fig. 8(A)c

GERT SIMULATION PROJECT 35 BY CITY ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	22.8600	13.5474	1000.	6.0000	76.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	67	28	32	27	30
			32	34	38	32	29	30	43	39	27	23	26
			28	25	31	20	25	18	27	24	26	239	

Fig. 8(A)c

GERT SIMULATION PROJECT 36 BY CITY ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	26.2450	14.4720	1000.	7.0000	107.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	0	46	23	30	21
			23	27	29	31	23	27	33	35	25	32	32
			23	26	19	35	29	25	18	27	26	335	

Fig. 8(A)c

GERT SIMULATION PROJECT 37 BY CITY ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	29.8170	15.3379	1000.	8.0000	108.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES									
9	1.00	1.00	0	0	0	0	0	0	0	0	36	12	18
			26	23	16	17	34	26	19	27	31	35	26
			26	24	21	23	27	22	21	28	23	439	

Fig. 8(A)c

GERT SIMULATION PROJECT 38 BY CITY ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	34.0300	16.7128	1000.	9.0000	124.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	0	0	0	18	15
			6	15	16	14	24	27	13	22	26	34	37
			22	25	26	26	18	25	19	17	19	535	

Fig. 8(A)c

GERT SIMULATION PROJECT 39 BY CITY ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	37.7050	17.6385	1000.	10.0000	114.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	0	0	0	0	11
			7	8	12	10	18	17	17	22	18	20	23
			18	20	27	16	25	27	28	16	23	617	

Fig. 8(A)c

TRANSITION MATRIX OF THE RETAIL TRADING ZONE

THE OBSERVED UNITS AND SAMPLE SIZES

	1	2	3
1	0.8292000E C0	0.1708000E 00	0.1000000E 01
2	0.6617000E 00	0.3383000E 00	0.1000000E 01
3	0.5121000E 00	0.4879000E 00	0.1000000E 01
4	0.4956999E 00	0.5043000E 00	0.1000000E 01
5	0.5C67CCCE 00	0.4933000E 00	0.1000000E 01

AS YOU DESIRED,COLUMN, 2IS DROPPED IN FORMING THE THE SYSTEM.
READ THE FOLLOWING MATRICES IN THE SEQUENCE 1 , 2
THE WEIGHT MATRIX IS DERIVED FROM THE MLE WHERE Z=1 FORDIAGONALS AND Z=0 OTHERWISE.

UNRESTRICTED ESTIMATOR OF THE TRANSITION MATRIX

0.71894467	0.28105533
0.25466180	0.74533820

Eig. 9(A)a

THE PRIDICTED PROPORTIONS

	1	2
2	0.6388506E 00	0.3611493E 00
3	0.5614545E 00	0.4385453E 00
4	0.4923294E 00	0.5076705E 00
5	0.4847514E 00	0.5152484E 00
6	0.4898342E 00	0.5101656E 00

SUM OF SQUARED ERROR	MEAN SQUARED ERROR	CHI SQUAREVALUE	MODIFIED CHI SQUARE
0.006902114	0.000862764	0.014129996	0.014054213

THE UNRESTRICTED ESTIMATOR IS PERFECT.

RECURSIVE DIFFERENCE=' 0.004433
RECURSIVE OUTPUT K= 2

UNRESTRICTED ESTIMATOR OF THE TRANSITION MATRIX

0.71774554	0.28225446
0.25574094	C.74425906

Fig. 9(A)a

GERT SIMULATION PROJECT 40 BY TRADING ZONE
DATE 4/ 20/ 1974

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 9
NUMBER OF SOURCE NODES IS 1
NUMBER OF SINK NODES IS 1
NUMBER OF NODES TO REALIZE THE NETWORK IS 1
STATISTICS COLLECTED ON 1 NODES
NUMBER OF PARAMETER SETS IS 11
INITIAL RANDOM NUMBER IS 7777777 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	0	9999	D		
3	1	1	P		
4	1	1	P		
5	1	1	D		
6	1	1	D		
7	1	1	D		
8	1	9999	D		
9	1	9999	D		A

SOURCE NODE NUMBERS
2

SINK NODE NUMBERS
9

Fig. 9(A)b

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0
8	1.0000	0.0	0.0	0.0
9	1.0000	0.0	0.0	0.0
10	1.0000	0.0	0.0	0.0
11	1.0000	0.0	0.0	0.0

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	1	0	0	1.0000
3	5	8	1	0	0	0.7180
3	4	9	1	0	0	0.2820
4	6	10	1	0	0	0.7440
4	7	11	1	0	0	0.2560
5	3	2	1	0	0	1.0000
5	8	3	1	0	0	1.0000
6	4	4	1	0	0	1.0000
7	8	5	1	0	0	1.0000
7	3	7	1	0	0	1.0000
8	9	6	1	0	0	1.0000

Fig. 9(A)b

GERT SIMULATION PROJECT 40 BY TRADING ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	2.0970	2.4730	1000.	1.0000	20.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	726	55	43	58	36	25	19	12	3	3
			3	6	3	0	1	2	0	3	1	1	0
			0	0	0	0	0	0	0	0	0	0	

Fig. 9(A)c

GERT SIMULATION PROJECT 41 BY TRADING ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	4.0120	3.1774	1000.	2.0000	22.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES									
9	1.00	1.00	0	0	526	96	87	74	62	29	27	27	20
			11	10	11	5	4	2	1	1	4	1	1
			1	0	0	0	0	0	0	0	0	0	

Fig. 9(A)c

GERT SIMULATION PROJECT 42 BY TRADING ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	6.2620	4.1519	1000.	3.0000	28.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	383	91	92	85	67	52	43	40
			39	20	19	17	11	5	8	6	7	6	3
			1	1	1	1	1	0	1	0	0	0	

Fig. 9(A)c

GERT SIMULATION PROJECT 43 BY TRADING ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE/TYPE
9	1.0000	8.3760	4.7985	1000.	4.0000	34.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	274	87	93	79	83	71	52
			56	42	19	29	26	18	16	8	5	16	5
			4	8	2	1	0	1	1	1	1	2	

Fig. 9(A)c

GERT SIMULATION PROJECT 44 BY TRADING ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT.	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	10.0460	5.0738	1000.	5.0000	39.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	229	77	86	79	68	81
			63	64	44	40	36	23	23	22	10	10	9
			6	8	4	6	2	4	1	0	2	3	

Fig. 9(A)c

GERT SIMULATION PROJECT 45 BY TRADING ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	12.3060	5.8950	1000.	6.0000	40.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	160	79	79	73	61
			81	71	64	49	46	35	31	26	33	12	20
			19	7	8	9	7	11	2	1	3	13	

Fig. 9(A)c

GERT SIMULATION PROJECT 46 BY TRADING ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	14.6990	8.4656	1000.	7.0000	46.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	0	100	70	66	58
			81	81	58	53	53	59	41	36	46	30	29
			15	21	19	14	11	12	11	7	4	25	

Fig. 9(A)c

GERT SIMULATION PROJECT 47 BY TRADING ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	16.8620	7.0108	1000.	8.0000	48.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES									
9	1.00	1.00	0	0	0	0	0	0	0	0	71	56	61
			69	70	70	57	51	55	47	43	41	44	33
			29	21	33	24	30	12	14	14	12	43	

Fig. 9(A)c

GERT SIMULATION PROJECT 48 BY TRADING ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	18.5890	7.1269	1000.	9.0000	47.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	0	0	0	62	46
			42	58	60	61	58	63	64	53	57	51	39
			34	32	28	30	21	24	14	16	17	70	

Fig. 9(A)c

GERT SIMULATION PROJECT 49 BY TRADING ZONE
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	20.4720	7.6707	1000.	10.0000	52.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.CC	1.00	0	0	0	0	0	0	0	0	0	0	47
			37	42	54	60	68	54	53	63	40	60	35
			51	51	31	33	26	29	16	18	20	112	

Fig. 9(A)c

TRANSITION MATRIX OF ESSEX COUNTY

THE OBSERVED UNITS AND SAMPLE SIZES

	1	2	3
1	0.7199000E 00	0.2801000E 00	0.1000000E 01
2	0.5560000E 00	0.4439999E 00	0.1000000E 01
3	0.6460000E 00	0.3540000E 00	0.1000000E 01
4	0.6355000E 00	0.3645000E 00	0.1000000E 01
5	0.6553000E 00	0.3447000E 00	0.1000000E 01

AS YOU DESIRED,COLUMN, 2IS DROPPED IN FORMING THE THE SYSTEM.
READ THE FOLLOWING MATRICES IN THE SEQUENCE 1 2
THE WEIGHT MATRIX IS DERIVED FROM THE MLE WHERE Z=1 FORDIAGONALS AND Z=0 OTHERWISE.

UNRESTRICTED ESTIMATOR OF THE TRANSITION MATRIX

0.43095815	0.56904185
0.96537751	0.03462249

Fig. 10(A)a

THE PRIDICTED PROPORTIONS

	1	2
2	0.5802460E 00	0.4197538E 00
3	0.6674563E 00	0.3325436E 00
4	0.6195678E 00	0.3804321E 00
5	0.6251548E 00	0.3748450E 00
6	0.6146193E 00	0.3853805E 00

SUM OF SQUARED ERROR	MEAN SQUARED ERROR	CHI SQUAREVALUE	MODIFIED CHI SQUARE
0.004421618	0.000552702	0.009442601	0.009513337

THE UNRESTRICTED ESTIMATOR IS PERFECT.

RECURSIVE DIFFERENCE=' 0.004650
RECURSIVE OUTPUT K= 2

UNRESTRICTED ESTIMATOR OF THE TRANSITION MATRIX

0.43115973	0.56884027
0.96338606	0.03661394

Fig. 10(A)a

GERT SIMULATION PROJECT 50 BY ESSEX COUNTY
DATE 4/ 20/ 1974

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 9
NUMBER OF SOURCE NODES IS 1
NUMBER OF SINK NODES IS 1
NUMBER OF NODES TO REALIZE THE NETWORK IS 1
STATISTICS COLLECTED ON 1 NODES
NUMBER OF PARAMETER SETS IS 11
INITIAL RANDOM NUMBER IS 7777777 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	0	9999	D		
3	1	1	P		
4	1	1	P		
5	1	1	D		
6	1	1	D		
7	1	1	D		
8	1	9999	D		
9	1	9999	D		A

SOURCE NODE NUMBERS
2

SINK NODE NUMBERS
9

Fig. 10(A)b

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0
8	1.0000	0.0	0.0	0.0
9	1.0000	0.0	0.0	0.0
10	1.0000	0.0	0.0	0.0
11	1.0000	0.0	0.0	0.0

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	1	0	0	1.0000
3	4	9	1	0	0	0.5690
3	5	8	1	0	0	0.4310
4	7	11	1	0	0	0.9630
4	6	10	1	0	0	0.0370
5	3	2	1	0	0	1.0000
5	8	3	1	0	0	1.0000
6	4	4	1	0	0	1.0000
7	8	5	1	0	0	1.0000
7	3	7	1	0	0	1.0000
8	9	6	1	0	0	1.0000

Fig. 10(A)b

GERT SIMULATION PROJECT 50 BY ESSEX COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	1.5940	0.5267	1000.	1.0000	3.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	424	558	18	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	

Fig. 10(A)c

GERT SIMULATION PROJECT 51 BY ESSEX COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	3.1460	0.7370	1000.	2.0000	6.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	186	504	289	20	1	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	

Fig. 10(A)c

GERT SIMULATICN PROJECT 52 BY ESSEX COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	4.7510	0.9054	1000.	3.0000	8.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	82	305	406	195	11	1	0	0
			0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	

Fig. 10(A)c

GERT SIMULATION PROJECT 53 BY ESSEX COUNTY
DATE. 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE.
9	1.0000	6.3820	1.0560	1000.	4.0000	9.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	31	166	354	307	123	19	0
			0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	

Fig. 10(A)c

GERT SIMULATION PROJECT 54 BY ESSEX COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	7.9450	1.1495	1000.	5.0000	11.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	12	86	256	330	233	73
			10	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	

Fig. 10(A)c

GERT SIMULATION PROJECT 55 BY ESSEX COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	9.5340	1.2468	1000.	6.0000	13.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	5	42	164	269	294
			179	43	4	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	

Fig._10(A)c

GERT SIMULATION PROJECT 56 BY ESSEX COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	11.1220	1.3472	1000.	7.0000	16.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	0	3	15	90	221
			287	228	121	31	3	1	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	

Fig. 10(A)c.

GERT SIMULATION PROJECT 57 BY ESSEX COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	12.6960	1.4912	1000.	9.0000	17.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	0	0	0	13	51
			147	249	246	173	96	21	4	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	

Fig.. 10(A)c

GERT SIMULATION PROJECT 58 BY ESSEX COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	14.3390	1.6244	1000.	10.0000	19.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	0	0	0	0	6
			32	87	173	248	234	122	66	30	2	0	0
			0	0	0	0	0	0	0	0	0	0	

Fig. 10(A)c

GERT SIMULATION PROJECT 59 BY ESSEX COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	15.9790	1.6862	1000.	10.0000	21.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	0	0	0	0	1
			1	12	55	111	220	229	190	110	52	16	3
			0	0	0	0	0	0	0	0	0	0	

Fig. 10(A)c

TRANSITION MATRIX OF KENT COUNTY

THE OBSERVED UNITS AND SAMPLE SIZES

	1	2	3
1	0.3429000CE 00	0.6571000E 00	0.1000000E 01
2	0.2962000E 00	0.7038000E 00	0.1000000E 01
3	0.2665000E 00	0.7334000E CO	0.1000000E 01
4	0.2406000E 00	0.7594000E 00	0.1000000E 01
5	0.2354000E 00	0.7646000E 00	0.1000000E 01

AS YOU DESIRED,COLUMN, 2IS DROPPED IN FORMING THE THE SYSTEM.
REAC THE FOLLOWING MATRICES IN THE SEQUENCE 1 2
THE WEIGHT MATRIX IS DERIVED FROM THE MLE WHERE Z=1 FORDIAGONALS AND Z=0 OTHERWISE.

UNRESTRICTED ESTIMATOR OF THE TRANSITION MATRIX

0.70486414	0.29513586
0.08081621	0.91918379

Fig. 11(A)a

```
THE PRIDICTED PROPORTIONS

          1               2
 2   0.2948026E 00 0.7051972E 00
 3   0.2656971E 00 0.7343028E 00
 4   0.2471787E 00 C.7527212E 00
 5   0.2310448E 00 0.7689551E 00
 6   0.2278039E 00 0.7721959E 00

SUM OF SQUARED ERROR        MEAN SQUARED ERROR         CHI SQUAREVALUE        MODIFIED CHI SQUARE
     0.000131184                 0.000016398               0.000354040            0.000356896

THE UNRESTRICTED ESTIMATOR IS PERFECT.

RECURSIVE DIFFERENCE=' 0.001609
RECURSIVE OUTPUT K=     2

UNRESTRICTED ESTIMATOR OF THE TRANSITION MATRIX

    0.70434678      0.29565322
    0.08108807      0.91891193
```

Fig. 11(A)a

GERT SIMULATION PROJECT 50 BY KENT COUNTY
DATE 4/ 20/ 1974

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 9
NUMBER OF SOURCE NODES IS 1
NUMBER OF SINK NODES IS 1
NUMBER OF NODES TO REALIZE THE NETWORK IS 1
STATISTICS COLLECTED ON 1 NODES
NUMBER OF PARAMETER SETS IS 11
INITIAL RANDOM NUMBER IS 7777777 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	0	9999	D		
3	1	1	P		
4	1	1	P		
5	1	1	D		
6	1	1	D		
7	1	1	D		
8	1	9999	D		
9	1	9999	D		A

SOURCE NODE NUMBERS
2

SINK NODE NUMBERS
9

Fig. 11(A)b

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0
8	1.0000	0.0	0.0	0.0
9	1.0000	0.0	0.0	0.0
10	1.0000	0.0	0.0	0.0
11	1.0000	0.0	0.0	0.0

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	1	0	0	1.0000
3	5	8	1	0	0	0.7040
3	4	9	1	0	0	0.2960
4	6	10	1	0	0	0.9190
4	7	11	1	0	0	0.0810
5	3	2	1	0	0	1.0000
5	8	3	1	0	0	1.0000
6	4	4	1	0	0	1.0000
7	8	5	1	0	0	1.0000
7	3	7	1	0	0	1.0000
8	9	6	1	0	0	1.0000

Fig. 11(A)b

GERT SIMULATION PROJECT 60 BY KENT COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	4.6780	8.7563	1000.	1.0000	73.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	709	26	15	19	25	18	14	15	12	7
			15	8	9	8	9	6	5	9	7	2	4
			6	7	5	0	4	2	3	2	3	26	

Fig. 11(A)c

GERT SIMULATION PROJECT 61 BY KENT COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	9.0000	11.4834	1000.	2.0000	101.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	501	30	35	33	27	20	15	25	19
			24	19	17	15	12	22	14	12	11	16	9
			8	10	10	7	8	7	10	9	8	47	

Fig. 11(A)c

GERT SIMULATION PROJECT 62 BY KENT COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	13.4200	14.1397	1000.	3.0000	102.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	354	39	28	26	35	31	21	34
			26	26	30	23	23	11	22	19	14	12	15
			13	10	17	9	10	14	9	8	12	109	

Fig. 11(A)c

GERT SIMULATION PROJECT 63 BY KENT COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	17.7150	16.5553	1000.	4.0000	116.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	243	39	30	33	49	32	27
			22	24	28	23	19	30	19	28	16	20	20
			15	22	10	19	21	10	8	18	14	161	

Fig. 11(A)c

GERT SIMULATION PROJECT 64 BY KENT COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	21.9200	18.2933	1000.	5.0000	140.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	193	28	28	29	31	28
			27	30	24	26	20	27	25	19	22	26	18
			12	18	21	23	10	20	18	14	16	247	

Fig. 11(A)c

GERT SIMULATION PROJECT 65 BY KENT COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	27.1240	20.2796	1000.	6.0000	141.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	110	26	21	34	24
			27	22	30	28	26	23	26	33	19	17	28
			24	20	10	18	15	21	16	19	20	343	

Fig._11(A)c

GERT SIMULATION PROJECT 66 BY KENT COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	31.0000	21.3142	1000.	7.0000	146.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	0	90	23	16	21
			22	22	13	24	24	26	29	20	30	22	21
			26	12	21	15	17	25	18	26	18	419	

Fig.-11(A)c

GERT SIMULATION PROJECT 66 BY KENT COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	31.0000	21.3142	1000.	7.0000	146.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	0	90	23	16	21
			22	22	13	24	24	26	29	20	30	22	21
			26	12	21	15	17	25	18	26	18	419	

Fig. 11(A)c

GERT SIMULATION PROJECT 68 BY KENT COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	40.5490	24.2777	1000.	9.0000	148.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	0	0	0	43	17
			11	19	18	12	22	19	12	22	16	14	13
			17	18	24	22	22	15	18	21	14	591	

Fig. 11(A)c

GERT SIMULATION PROJECT 69 BY KENT COUNTY
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	45.2840	26.4302	1000.	10.0000	184.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	0	0	0	0	32
			7	10	13	16	14	12	12	10	15	22	13
			16	26	19	10	15	15	14	22	21	666	

Fig. 11(A)c

TRANSITION MATRIX OF LAMBTON COUNTY

THE OBSERVED UNITS AND SAMPLE SIZES

	1	2	3
1	0.1253999E 00	0.8746000E 00	0.1000000E 01
2	0.1071000E 00	0.8929000E 00	0.1000000E 01
3	0.9939998E-01	0.9006000E 00	0.1000000E 01
4	0.7559997E-01	0.9244000E 00	0.1000000E 01
5	0.6140000E-01	0.9385999E 00	0.1000000E 01

AS YOU DESIRED,COLUMN, 2IS DROPPED IN FORMING THE THE SYSTEM.
READ THE FOLLOWING MATRICES IN THE SEQUENCE 1 2
THE WEIGHT MATRIX IS DERIVED FROM THE MLE WHERE Z=1 FORDIAGONALS AND Z=0 OTHERWISE.

UNRESTRICTED ESTIMATOR OF THE TRANSITION MATRIX

0.94942534	0.05057466
-0.01247176	1.01247120

Fig. 12(A)a

```
THE FOLLOWINGS ARE QP SOLUTIONS

THIS PROBLEM HAS NO FEASIBLE SOLUTION.

NUMBER OF CYCLES REQUIRED=   2   OBJ.VALUE FOR LP OR  ROUNDING ERROR FOR QP=  0.2488375E-02

THE ML(GLS,MCS)ESTIMATOR OF THE TRANSITION MATRIX IS/
   0.83864129      0.16135871
   0.0             0.99999994
```

Fig. 12(A)a

7

THE PRIDICTED PROPORTIONS

	1	2
2	0.1051655E 00	0.8948343E 00
3	0.8981842E-01	0.9101814E 00
4	0.8336091E-01	0.9166389E 00
5	0.6340122E-01	0.9365986E 00
6	0.5149257E-01	0.9485072E 00

SUM OF SQUARED ERROR	MEAN SQUARED ERROR	CHI SQUAREVALUE	MODIFIED CHI SQUARE
0.000319570	0.000039946	0.002018462	0.001996047

RECURSIVE DIFFERENCE=' 0.000000

Fig. 12(A)b

GERT SIMULATION PROJECT 70 BY LAMBTON COUT
DATE 4/ 20/ 1974

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 5
NUMBER OF SOURCE NODES IS 1
NUMBER OF SINK NODES IS 1
NUMBER OF NODES TO REALIZE THE NETWORK IS 1
STATISTICS COLLECTED ON 1 NODES
NUMBER OF PARAMETER SETS IS 4
INITIAL RANDOM NUMBER IS 7777777 0.0

ODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	0	9999	D		
3	1	1	P		
4	1	1	D		A
5	1	9999	D		

SOURCE NODE NUMBERS
2

SINK NODE NUMBERS
5

Fig. 12(A)b

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0
3	1.0000	0.0	0.0	0.0
4	1.0000	0.0	0.0	0.0

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	1	0	0	1.0000
3	4	3	1	0	0	0.8390
3	5	4	1	0	0	0.1610
4	3	2	1	0	0	1.0000

Fig. 12(A)b

GERT SIMULATION PROJECT 70 BY LAMBTON COUT
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
5	1.0000	6.3490	6.0345	1000.	1.0000	50.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES									
5	1.00	1.00	0	157	142	121	100	64	58	64	42	33	40
			28	22	21	15	12	11	8	12	8	9	3
			4	2	4	3	3	3	0	3	1	7	

Fig. 12(A)c

```
TRANSITION MATRIX OF MIDDLESSEX COUNTY

THE OBSERVED UNITS AND SAMPLE SIZES

          1              2              3
 1  0.3700000E-02  0.9963000E 00  0.1000000E 01
 2  0.2500000E-02  0.9974999E 00  0.1000000E 01
 3  0.2300000E-02  0.9977000E 00  0.1000000E 01
 4  0.1900000E-02  0.9981000E 00  0.1000000E 01
 5  0.1500000E-02  0.9985000E 00  0.1000000E 01

AS YOU DESIRED,COLUMN, 2IS DROPPED IN FORMING THE THE SYSTEM.
READ THE FOLLOWING MATRICES IN THE SEQUENCE  1  2
 THE WEIGHT MATRIX IS DERIVED FROM THE MLE WHERE Z=1 FORDIAGONALS AND Z=0 OTHERWISE.

UNRESTRICTED ESTIMATOR OF THE TRANSITION MATRIX

   0.53817272      0.46182728
   0.00063550      0.99936455
```

Fig. 13(A)a

THE PRIDICTED PROPORTIONS

	1	2
2	0.2636552E-02	0.9973634E 00
3	0.1996667E-02	0.9980032E 00
4	0.1890020E-02	0.9981099E 00
5	0.1676725E-02	0.9983233E 00
6	0.1463430E-02	0.9985365E 00

SUM OF SQUARED ERROR	MEAN SQUARED ERROR	CHI SQUAREVALUE	MODIFIED CHI SQUARE
0.000000284	0.000000035	0.000071976	0.000068479

THE UNRESTRICTED ESTIMATOR IS PERFECT.

RECURSIVE DIFFERENCE=' 0.001100
RECURSIVE OUTPUT K= 3

UNRESTRICTED ESTIMATOR OF THE TRANSITION MATRIX

0.53387076	0.46612924
0.00066365	0.99933636

Fig. 13(A)a

GERT SIMULATION PROJECT 71 BY MIDDLESEX
DATE 4/ 20/ 1974

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 5
NUMBER OF SOURCE NODES IS 1
NUMBER OF SINK NODES IS 1
NUMBER OF NODES TO REALIZE THE NETWORK IS 1
STATISTICS COLLECTED ON 1 NODES
NUMBER OF PARAMETER SETS IS 4
INITIAL RANDOM NUMBER IS 7777777 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	0	9999	D		
3	1	1	P		
4	1	1	D		A
5	1	9999	D		

SOURCE NODE NUMBERS
2

SINK NODE NUMBERS
5

Fig. 13(A)b

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0
3	1.0000	0.0	0.0	0.0
4	1.0000	0.0	0.0	0.0

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	1	0	0	1.0000
3	4	3	1	0	0	0.5330
3	5	4	1	0	0	0.4670
4	3	2	1	0	0	1.0000

Fig. 13(A)b

GERT SIMULATION PROJECT 71 BY MIDDLESSEX
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
5	1.0000	2.1470	1.5887	1000.	1.0000	18.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
5	1.00	1.00	0	469	229	157	70	34	18	12	5	2	2
			1	0	0	0	0	0	0	1	0	0	0
			0	0	0	0	0	0	0	0	0	0	

Fig. 13(A)c

TRANSITION MATRIX FOR SOUTHERN ONTARIO

THE OBSERVED UNITS AND SAMPLE SIZES

	1	2	3
1	0.3819000E 00	0.6181000E 00	0.1000000E 01
2	0.3816000E 00	0.6184000E 00	0.1000000E 01
3	0.3110999E 00	0.6889000E 00	0.1000000E 01
4	0.3001000E 00	0.6999000E 00	0.1000000E 01
5	0.3042000E 00	0.6957999E 00	0.1000000E 01

AS YOU DESIRED,COLUMN, 2IS DROPPED IN FORMING THE THE SYSTEM.
READ THE FOLLOWING MATRICES IN THE SEQUENCE 1 2
THE WEIGHT MATRIX IS DERIVED FROM THE MLE WHERE Z=1 FORDIAGONALS AND Z=0 OTHERWISE.

UNRESTRICTED ESTIMATOR OF THE TRANSITION MATRIX

0.68453145	0.31546855
0.13427496	0.86572504

Fig. 14(A)a

```
THE PRIDICTED PROPORTIONS

            1                  2
  2   0.3460998E 00 0.6539000E 00
  3   0.3459283E 00 0.6540715E 00
  4   0.3056328E 00 0.6943671E 00
  5   0.2993455E 00 0.7006543E 00
  6   0.3016890E 00 0.6983109E 00

SUM OF SQUARED ERROR        MEAN SQUARED ERROR        CHI SQUAREVALUE        MODIFIED CHI SQUARE
     0.005054902                 0.000631863              0.011186332              0.011257496

THE UNRESTRICTED ESTIMATOR IS PERFECT.

RECURSIVE DIFFERENCE=  0.042624
RECURSIVE OUTPUT K=     2

UNRESTRICTED ESTIMATOR OF THE TRANSITION MATRIX

    0.69933957      0.30066043
    0.12784266      0.87215734
```

Fig.- 14(A)a

GERT SIMULATION PROJECT 30 BY S.ONTARIO
DATE 4/ 20/ 1974

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 9
NUMBER OF SOURCE NODES IS 1
NUMBER OF SINK NODES IS 1
NUMBER OF NODES TO REALIZE THE NETWORK IS 1
STATISTICS COLLECTED ON 1 NODES
NUMBER OF PARAMETER SETS IS 11
INITIAL RANDOM NUMBER IS 7777777 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	0	9999	D		
3	1	1	P		
4	1	1	P		
5	1	1	D		
6	1	1	D		
7	1	1	D		
8	1	9999	D		A
9	1	9999	D		

SOURCE NODE NUMBERS
2

SINK NODE NUMBERS
9

Fig. 14(A)b

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0
8	1.0000	0.0	0.0	0.0
9	1.0000	0.0	0.0	0.0
10	1.0000	0.0	0.0	0.0
11	1.0000	0.0	0.0	0.0

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	1	0	0	1.0000
3	5	8	1	0	0	0.6990
3	4	9	1	0	0	0.3010
4	6	10	1	0	0	0.8720
4	7	11	1	0	0	0.1280
5	3	2	1	0	0	1.0000
5	8	3	1	0	0	1.0000
6	4	4	1	0	0	1.0000
7	8	5	1	0	0	1.0000
7	3	7	1	0	0	1.0000
8	9	6	1	0	0	1.0000

Fig. 14(A)b

GERT SIMULATION PROJECT 30 BY S.ONTARIO
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	3.4260	5.5614	1000.	1.0000	39.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	713	29	27	23	24	29	20	17	21	9
			8	11	8	8	5	4	6	5	6	0	3
			2	2	4	1	2	0	1	1	3	8	

Fig. 14(A)c

GERT SIMULATION PROJECT 30 BY S.ONTARIO
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	6.9490	7.7034	1000.	2.0000	55.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	481	57	42	38	41	32	17	40	24
			21	25	23	18	22	14	8	9	11	13	3
			9	6	6	6	2	4	1	3	3	21	

Fig. 14(A)c

GERT SIMULATION PROJECT 30 BY S.ONTARIO
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	10.0450	9.5180	1000.	3.0000	71.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	347	55	51	44	50	46	31	49
			27	28	20	23	22	24	16	15	15	8	18
			11	9	7	14	4	3	4	5	7	47	

Fig. 14(A)c

GERT SIMULATION PROJECT 30 BY S.ONTARIO
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	13.3130	10.6812	1000.	4.0000	70.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	234	50	44	67	43	38	45
			34	31	36	28	30	36	31	20	29	12	14
			20	15	17	9	12	7	7	8	8	75	

Fig. 14(A)c

GERT SIMULATION PROJECT 30 BY S.ONTARIO
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	16.7840	11.8973	1000.	5.0000	75.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	170	48	32	36	40	39
			30	47	46	45	25	37	40	31	31	21	21
			19	21	18	16	17	13	10	13	9	125	

Fig. 14(A)c

GERT SIMULATION PROJECT 30 BY S.ONTARIO
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	19.7060	12.7856	1000.	6.0000	83.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	131	34	29	45	37
			41	29	40	29	38	28	39	46	29	31	32
			13	23	22	19	20	26	20	11	20	168	

Fig. 14(A)c

GERT SIMULATION PROJECT 30 BY S.ONTARIO
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	22.9730	13.7111	1000.	7.0000	83.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES									
9	1.00	1.00	0	0	0	0	0	0	0	98	37	27	35
			27	30	26	38	38	29	32	39	34	27	30
			27	25	24	21	21	23	27	20	17	248	

Fig. 14(A)c

GERT SIMULATION PROJECT 30 BY S.ONTAFIO
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	26.4900	15.0748	1000.	8.0000	98.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	0	0	58	31	13
			37	25	23	34	36	31	36	36	42	27	28
			43	16	22	19	22	19	32	18	27	325	

Fig._14(A)c

GERT SIMULATION PROJECT 30 BY S.ONTARIO
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	29.9170	15.4978	1000.	9.0000	104.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	0	0	0	40	16
			18	26	28	21	30	31	26	19	26	33	32
			23	36	29	23	27	24	23	29	25	415	

Fig. 14(A)c

GERT SIMULATION PROJECT 30 BY S.ONTARIO
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	33.0230	16.7316	1000.	10.0000	121.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
9	1.00	1.00	0	0	0	0	0	0	0	0	0	0	33
			14	23	17	22	23	22	21	21	25	19	29
			19	22	27	27	27	26	29	29	38	487	

Fig. 14(A)c

GERT SIMULATICN PROJECT 20 BY DETROIT 20
DATE 4/ 20/ 1974

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 8
NUMBER OF SOURCE NODES IS 1
NUMBER OF SINK NODES IS 1
NUMBER OF NODES TO REALIZE THE NETWORK IS 1
STATISTICS COLLECTED CN 1 NODES
NUMBER OF PARAMETER SETS IS 16
INITIAL RANDOM NUMBER IS 1783297 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	1	1	P		
3	1	1	P		
4	1	1	P		
5	1	1	P		
6	1	1	P		
7	1	1	P		
8	1	1	D		A

SOURCE NODE NUMBERS
2

SINK NODE NUMBERS
8

Fig. 15(A)b

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	1.0000	0.0	0.0	0.0
2	1.0000	0.0	0.0	0.0
3	1.0000	0.0	0.0	0.0
4	1.0000	0.0	0.0	0.0
5	1.0000	0.0	0.0	0.0
6	1.0000	0.0	0.0	0.0
7	1.0000	0.0	0.0	0.0
8	1.0000	0.0	0.0	0.0
9	1.0000	0.0	0.0	0.0
10	1.0000	0.0	0.0	0.0
11	1.0000	0.0	0.0	0.0
12	1.0000	0.0	0.0	0.0
13	1.0000	0.0	0.0	0.0
14	1.0000	0.0	0.0	0.0
15	1.0000	0.0	0.0	0.0
16	1.0000	0.0	0.0	0.0

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	1	0	0	C.7400
2	8	3	1	0	0	0.2450
2	6	2	1	0	0	0.0150
3	4	4	1	0	0	0.8410
3	8	6	1	0	0	0.1450
3	7	5	1	0	0	0.0140
4	4	7	1	0	0	0.9280
4	8	9	1	0	0	0.0590
4	7	8	1	0	0	0.0130
5	6	10	1	0	0	0.8570
5	8	11	1	0	0	0.1430
6	7	12	1	0	0	0.8330
6	8	13	1	0	0	0.1670
7	7	15	1	0	0	0.9200
7	4	14	1	0	0	0.0430
7	8	16	1	0	0	0.0370

Fig. 15(A)b

GERT SIMULATION PROJECT 20 BY DETROIT 20
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
8	1.0000	12.3810	15.4289	1000.	1.0000	111.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
8	1.00	5.00	0	465	150	97	83	57	41	28	17	19	10
			9	8	6	1	1	1	1	0	1	2	1
			1	1	0	0	0	0	0	0	0	0	

Fig. 15(A)c

```
GERT SIMULATION PROJECT  20 BY DETROIT 21
DATE  4/ 20/ 1974

**NETWORK DESCRIPTION**

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS      8
NUMBER OF SOURCE NODES IS     1
NUMBER OF SINK NODES IS     1
NUMBER OF NODES TO REALIZE THE NETWORK IS     1
STATISTICS COLLECTED ON     1 NODES
NUMBER OF PARAMETER SETS IS   16
INITIAL RANDOM NUMBER IS 1783297     0.0
```

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	1	1	P		
3	1	1	P		
4	1	1	P		
5	1	1	P		
6	1	1	P		
7	1	1	P		
8	1	1	D		

```
SOURCE NODE NUMBERS
     3

SINK NODE NUMBERS
     8
```

Fig. 16(A)b

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	1.0000	0.0	0.0	0.0
2	1.0000	0.0	0.0	0.0
3	1.0000	0.0	0.0	0.0
4	1.0000	0.0	0.0	0.0
5	1.0000	0.0	0.0	0.0
6	1.0000	0.0	0.0	0.0
7	1.0000	0.0	0.0	0.0
8	1.0000	0.0	0.0	0.0
9	1.0000	0.0	0.0	0.0
10	1.0000	0.0	0.0	0.0
11	1.0000	0.0	0.0	0.0
12	1.0000	0.0	0.0	0.0
13	1.0000	0.0	0.0	0.0
14	1.0000	0.0	0.0	0.0
15	1.0000	0.0	0.0	0.0
16	1.0000	0.0	0.0	0.0

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	1	0	0	0.7400
2	8	3	1	0	0	0.2450
2	6	2	1	0	0	0.0150
3	4	4	1	0	0	0.8410
3	8	6	1	0	0	0.1450
3	7	5	1	0	0	0.0140
4	4	7	1	0	0	0.9280
4	8	9	1	0	0	0.0590
4	7	8	1	0	0	0.0130
5	6	10	1	0	0	0.8570
5	8	11	1	0	0	0.1430
6	7	12	1	0	0	0.8330
6	8	13	1	0	0	0.1670
7	7	15	1	0	0	0.9200
7	4	14	1	0	0	0.0430
7	8	16	1	0	0	0.0370

Fig.16(A)b

GERT SIMULATION PROJECT 20 BY DETROIT'21
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
8	1.0000	15.0490	16.1875	1000.	1.0000	112.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
8	1.00	5.00	0	345	174	128	100	65	55	34	16	26	14
			11	10	10	1	2	1	0	2	1	1	2
			1	1	0	0	0	0	0	0	0	0	

Fig. 16(A)b

GERT SIMULATION PROJECT 20 BY DETROIT 22
DATE 4/ 20/ 1974

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 8
NUMBER OF SOURCE NODES IS 1
NUMBER OF SINK NODES IS 1
NUMBER OF NODES TO REALIZE THE NETWORK IS 1
STATISTICS COLLECTED ON 1 NODES
NUMBER OF PARAMETER SETS IS 16
INITIAL RANDOM NUMBER IS 1783297 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	1	1	P		
3	1	1	P		
4	1	1	P		
5	1	1	P		
6	1	1	P		
7	1	1	P		
8	1	1	D		A

SOURCE NODE NUMBERS
4

SINK NODE NUMBERS
8

Fig. 17(A)b

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	1.0000	0.0	0.0	0.0
2	1.0000	0.0	0.0	0.0
3	1.0000	0.0	0.0	0.0
4	1.0000	0.0	0.0	0.0
5	1.0000	0.0	0.0	0.0
6	1.0000	0.0	0.0	0.0
7	1.0000	0.0	0.0	0.0
8	1.0000	0.0	0.0	0.0
9	1.0000	0.0	0.0	0.0
10	1.0000	0.0	0.0	0.0
11	1.0000	0.0	0.0	0.0
12	1.0000	0.0	0.0	0.0
13	1.0000	0.0	0.0	0.0
14	1.0000	0.0	0.0	0.0
15	1.0000	0.0	0.0	0.0
16	1.0000	0.0	0.0	0.0

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	1	0	0	0.7400
2	8	3	1	0	0	0.2450
2	6	2	1	0	0	0.0150
3	4	4	1	0	0	0.8410
3	8	6	1	0	0	0.1450
3	7	5	1	0	0	0.0140
4	4	7	1	0	0	0.9280
4	8	9	1	0	0	0.0590
4	7	8	1	0	0	0.0130
5	6	10	1	0	0	0.8570
5	8	11	1	0	0	0.1430
6	7	12	1	0	0	0.8330
6	8	13	1	0	0	0.1670
7	7	15	1	0	0	0.9200
7	4	14	1	0	0	0.0430
7	8	16	1	0	0	0.0370

Fig. 17(A)c

GERT SIMULATION PROJECT 20 BY DETROIT 22
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
8	1.0000	16.6720	16.6532	1000.	1.0000	120.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
8	1.00	5.00	0	274	196	139	106	73	63	36	18	31	15
			11	11	11	3	3	1	0	2	1	1	2
			1	1	1	0	0	0	0	0	0	0	

Fig. 17(A)c

GERT SIMULATION PROJECT 20 BY DETROIT 23
DATE 4/ 20/ 1974

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 8
NUMBER OF SOURCE NODES IS 1
NUMBER OF SINK NODES IS 1
NUMBER OF NODES TO REALIZE THE NETWORK IS 1
STATISTICS COLLECTED ON 1 NODES
NUMBER OF PARAMETER SETS IS 16
INITIAL RANDOM NUMBER IS 1783297 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	1	1	P		
3	1	1	P		
4	1	1	P		
5	1	1	P		
6	1	1	P		
7	1	1	P		
8	1	1	D		A

SOURCE NODE NUMBERS
5

SINK NODE NUMBERS
8

Fig. 18(A)b

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	1.0000	0.0	0.0	0.0
2	1.0000	0.0	0.0	0.0
3	1.0000	0.0	0.0	0.0
4	1.0000	0.0	0.0	0.0
5	1.0000	0.0	0.0	0.0
6	1.0000	0.0	0.0	0.0
7	1.0000	0.0	0.0	0.0
8	1.0000	0.0	0.0	0.0
9	1.0000	0.0	0.0	0.0
10	1.0000	0.0	0.0	0.0
11	1.0000	0.0	0.0	0.0
12	1.0000	0.0	0.0	0.0
13	1.0000	0.0	0.0	0.0
14	1.0000	0.0	0.0	0.0
15	1.0000	0.0	0.0	0.0
16	1.0000	0.0	0.0	0.0

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	1	0	0	0.7400
2	8	3	1	0	0	0.2450
2	6	2	1	0	0	0.0150
3	4	4	1	0	0	0.8410
3	8	6	1	0	0	0.1450
3	7	5	1	0	0	0.0140
4	4	7	1	0	0	0.9280
4	8	9	1	0	0	0.0590
4	7	8	1	0	0	0.0130
5	6	10	1	0	0	0.8570
5	8	11	1	0	0	0.1430
6	7	12	1	0	0	0.8330
6	8	13	1	0	0	0.1670
7	7	15	1	0	0	0.9200
7	4	14	1	0	0	0.0430
7	8	16	1	0	0	0.0370

Fig. 18(A)b

GERT SIMULATION PROJECT 20 BY DETROIT 23
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
8	1.0000	16.6530	18.5878	1000.	1.0000	145.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
8	1.00	5.00	0	373	117	107	91	80	59	39	32	27	16
			13	9	10	10	4	2	3	1	0	1	3
			1	1	0	0	0	0	0	1	0	0	

Fig. 18(A)b

GERT SIMULATION PROJECT 20 BY DETROIT 24
DATE 4/ 20/ 1974

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 8
NUMBER OF SOURCE NODES IS 1
NUMBER OF SINK NODES IS 1
NUMBER OF NODES TO REALIZE THE NETWORK IS 1
STATISTICS COLLECTED ON 1 NODES
NUMBER OF PARAMETER SETS IS 16
INITIAL RANDOM NUMBER IS 1783297 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	1	1	P		
3	1	1	P		
4	1	1	P		
5	1	1	P		
6	1	1	P		
7	1	1	P		A
8	1	1	D		

SOURCE NODE NUMBERS
6

SINK NODE NUMBERS
8

Fig. 19(A)b

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	1.0000	0.0	0.0	0.0
2	1.0000	0.0	0.0	0.0
3	1.0000	0.0	0.0	0.0
4	1.0000	0.0	0.0	0.0
5	1.0000	0.0	0.0	0.0
6	1.0000	0.0	0.0	0.0
7	1.0000	0.0	0.0	0.0
8	1.0000	0.0	0.0	0.0
9	1.0000	0.0	0.0	0.0
10	1.0000	0.0	0.0	0.0
11	1.0000	0.0	0.0	0.0
12	1.0000	0.0	0.0	0.0
13	1.0000	0.0	0.0	0.0
14	1.0000	0.0	0.0	0.0
15	1.0000	0.0	0.0	0.0
16	1.0000	0.0	0.0	0.0

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	1	0	0	0.7400
2	8	3	1	0	0	0.2450
2	6	2	1	0	0	0.0150
3	4	4	1	0	0	0.8410
3	8	6	1	0	0	0.1450
3	7	5	1	0	0	0.0140
4	4	7	1	0	0	0.9280
4	8	9	1	0	0	0.0590
4	7	8	1	0	0	0.0130
5	6	10	1	0	0	0.8570
5	8	11	1	0	0	0.1430
6	7	12	1	0	0	0.8330
6	8	13	1	0	0	0.1670
7	7	15	1	0	0	0.9200
7	4	14	1	0	0	0.0430
7	8	16	1	0	0	0.0370

Fig. 19(A)b

GERT SIMULATION PROJECT 20 BY DETROIT 24
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
8	1.0000	18.2670	18.9865	1000.	1.0000	144.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
8	1.00	5.00	0	300	131	120	105	89	69	44	30	30	18
			10	14	11	10	2	1	5	2	1	1	3
			1	1	0	0	1	0	0	1	0	0	

Fig. 19(A)c

GERT SIMULATION PROJECT 20 BY DETROIT 25
DATE 4/ 20/ 1974

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 8
NUMBER OF SOURCE NODES IS 1
NUMBER OF SINK NODES IS 1
NUMBER OF NODES TO REALIZE THE NETWORK IS 1
STATISTICS COLLECTED ON 1 NODES
NUMBER OF PARAMETER SETS IS 16
INITIAL RANDOM NUMBER IS 1783297 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	1	1	P		
3	1	1	P		
4	1	1	P		
5	1	1	P		
6	1	1	P		
7	1	1	P		
8	1	1	D		A

SOURCE NODE NUMBERS
7

SINK NODE NUMBERS
8

Fig. 20(A)b

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	1.0000	0.0	0.0	0.0
2	1.0000	0.0	0.0	0.0
3	1.0000	0.0	0.0	0.0
4	1.0000	0.0	0.0	0.0
5	1.0000	0.0	0.0	0.0
6	1.0000	0.0	0.0	0.0
7	1.0000	0.0	0.0	0.0
8	1.0000	0.0	0.0	0.0
9	1.0000	0.0	0.0	0.0
10	1.0000	0.0	0.0	0.0
11	1.0000	0.0	0.0	0.0
12	1.0000	0.0	0.0	0.0
13	1.0000	0.0	0.0	0.0
14	1.0000	0.0	0.0	0.0
15	1.0000	0.0	0.0	0.0
16	1.0000	0.0	0.0	0.0

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	1	0	0	0.7400
2	8	3	1	0	0	0.2450
2	6	2	1	0	0	0.0150
3	4	4	1	0	0	0.8410
3	8	6	1	0	0	0.1450
3	7	5	1	0	0	0.0140
4	4	7	1	0	0	0.9280
4	8	9	1	0	0	0.0590
4	7	8	1	0	0	0.0130
5	6	10	1	0	0	0.8570
5	8	11	1	0	0	0.1430
6	7	12	1	0	0	0.8330
6	8	13	1	0	0	0.1670
7	7	15	1	0	0	0.9200
7	4	14	1	0	0	0.0430
7	8	16	1	0	0	0.0370

Fig. 26(A)b

GERT SIMULATION PROJECT 20 BY DETROIT 25
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
8	1.0000	21.0530	19.3133	1000.	1.0000	145.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
8	1.00	5.00	0	188	164	132	117	103	76	50	33	38	27
			10	15	11	12	3	2	5	3	2	2	3
			1	0	0	0	1	1	0	1	0	0	

Fig. 20(A)c

GERT SIMULATION PROJECT 27 BY DETROIT 27
DATE 4/ 20/ 1974

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 9
NUMBER OF SOURCE NODES IS 1
NUMBER OF SINK NODES IS 1
NUMBER OF NODES TO REALIZE THE NETWORK IS 1
STATISTICS COLLECTED ON 1 NODES
NUMBER OF PARAMETER SETS IS 22
INITIAL RANDOM NUMBER IS 1783297 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
9	0	9999	P		
2	1	1	P		
3	1	1	P		
4	1	1	P		
5	1	1	P		
6	1	1	P		
7	1	1	P		
8	1	9999	D		A

SOURCE NODE NUMBERS
9

SINK NODE NUMBERS
8

Fig. 21(A)b

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	1.0000	0.0	0.0	0.0
2	1.0000	0.0	0.0	0.0
3	1.0000	0.0	0.0	0.0
4	1.0000	0.0	0.0	0.0
5	1.0000	0.0	0.0	0.0
6	1.0000	0.0	0.0	0.0
7	1.0000	0.0	0.0	0.0
8	1.0000	0.0	0.0	0.0
9	1.0000	0.0	0.0	0.0
10	1.0000	0.0	0.0	0.0
11	1.0000	0.0	0.0	0.0
12	1.0000	0.0	0.0	0.0
13	1.0000	0.0	0.0	0.0
14	1.0000	0.0	0.0	0.0
15	1.0000	0.0	0.0	0.0
16	1.0000	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0
21	0.0	0.0	0.0	0.0
22	0.0	0.0	0.0	0.0

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	1	0	0	0.7400
2	8	3	1	0	0	0.2450
2	6	2	1	0	0	0.0150
3	4	4	1	0	0	0.8410
3	8	6	1	0	0	0.1450
3	7	5	1	0	0	0.0140
4	4	7	1	0	0	0.9280
4	9	9	1	0	0	0.0590
4	7	8	1	0	0	0.0130
5	6	10	1	0	0	0.8570
5	8	11	1	0	0	0.1430
6	7	12	1	0	0	0.8330
6	8	13	1	0	0	0.1670
7	7	15	1	0	0	0.9200
7	4	14	1	0	0	0.0430
7	8	16	1	0	0	0.0370
9	4	19	1	0	0	0.5070
9	2	17	1	0	0	0.2080
9	7	22	1	0	0	0.1460
9	3	18	1	0	0	0.1280
9	6	21	1	0	0	0.0080
9	5	20	1	0	0	0.0030

Fig. 21(A)b

GERT SIMULATION PROJECT 27 BY DETROIT 27
DATE 4/ 20/ 1974

FINAL RESULTS FOR 1000 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	= OF OBS.	MIN.	MAX.	NODE TYPE
8	1.0000	16.0480	16.7845	1000.	1.0000	144.0000	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
8	1.00	5.00	0	310	177	132	103	77	49	33	27	30	14
			16	7	9	3	3	2	1	2	0	1	1
			1	1	0	0	0	0	0	1	0	0	

Fig. 21(A)c

APPENDIX (B)

FIGURES AND GERT III REPRESENTATIONS

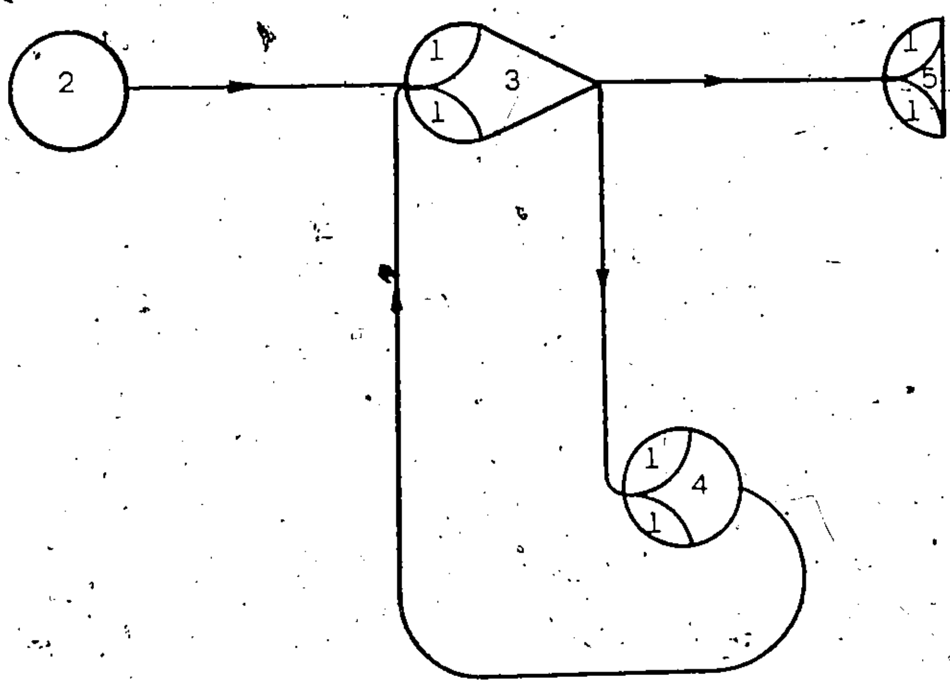

Fig. 1(B) - GERT III representation for Lambton and Middlesex Counties Markov models.

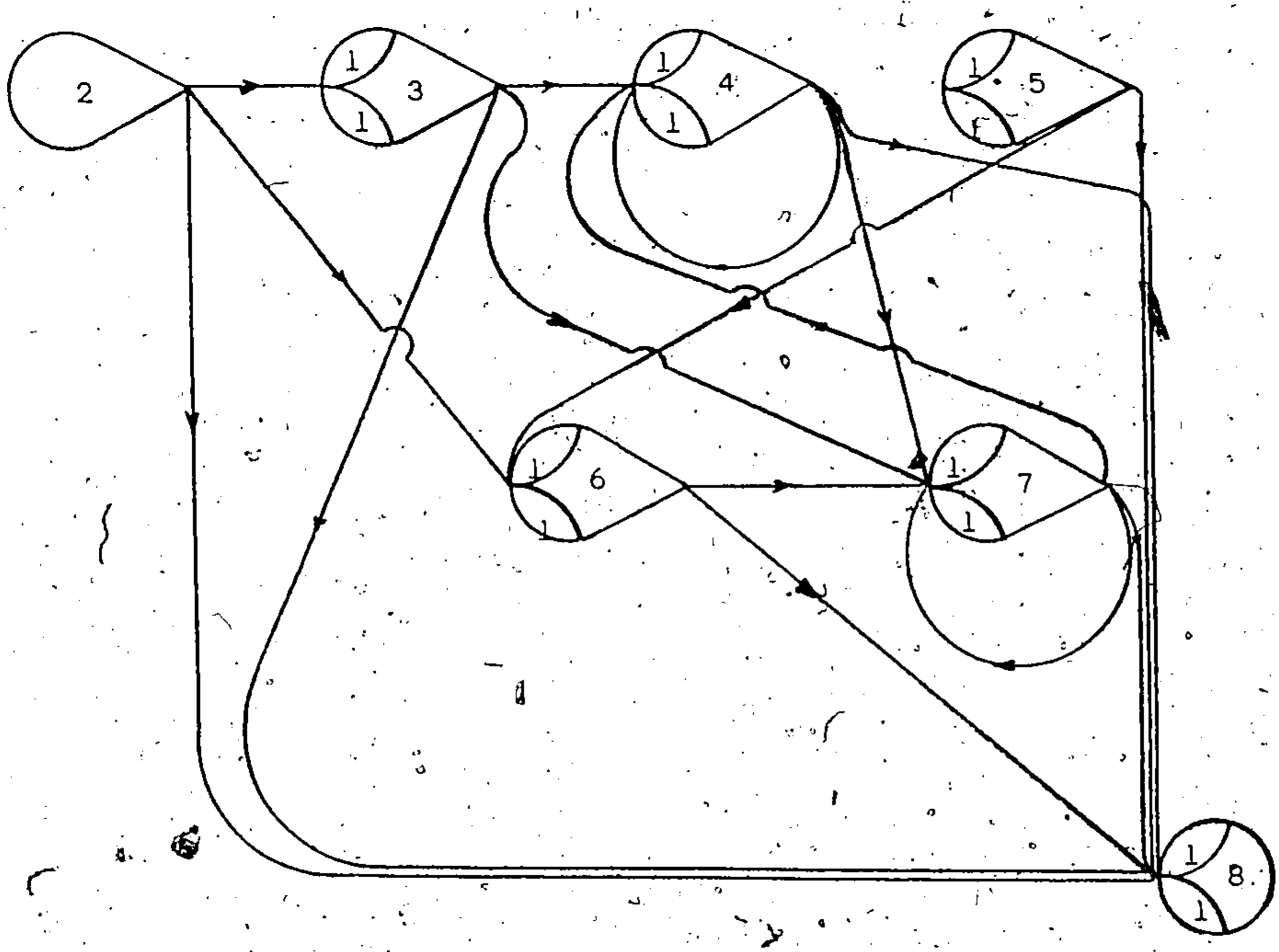

Fig. 2(B) - GERT III elements representing a subscriber to The Detroit Evening News who is in state one.

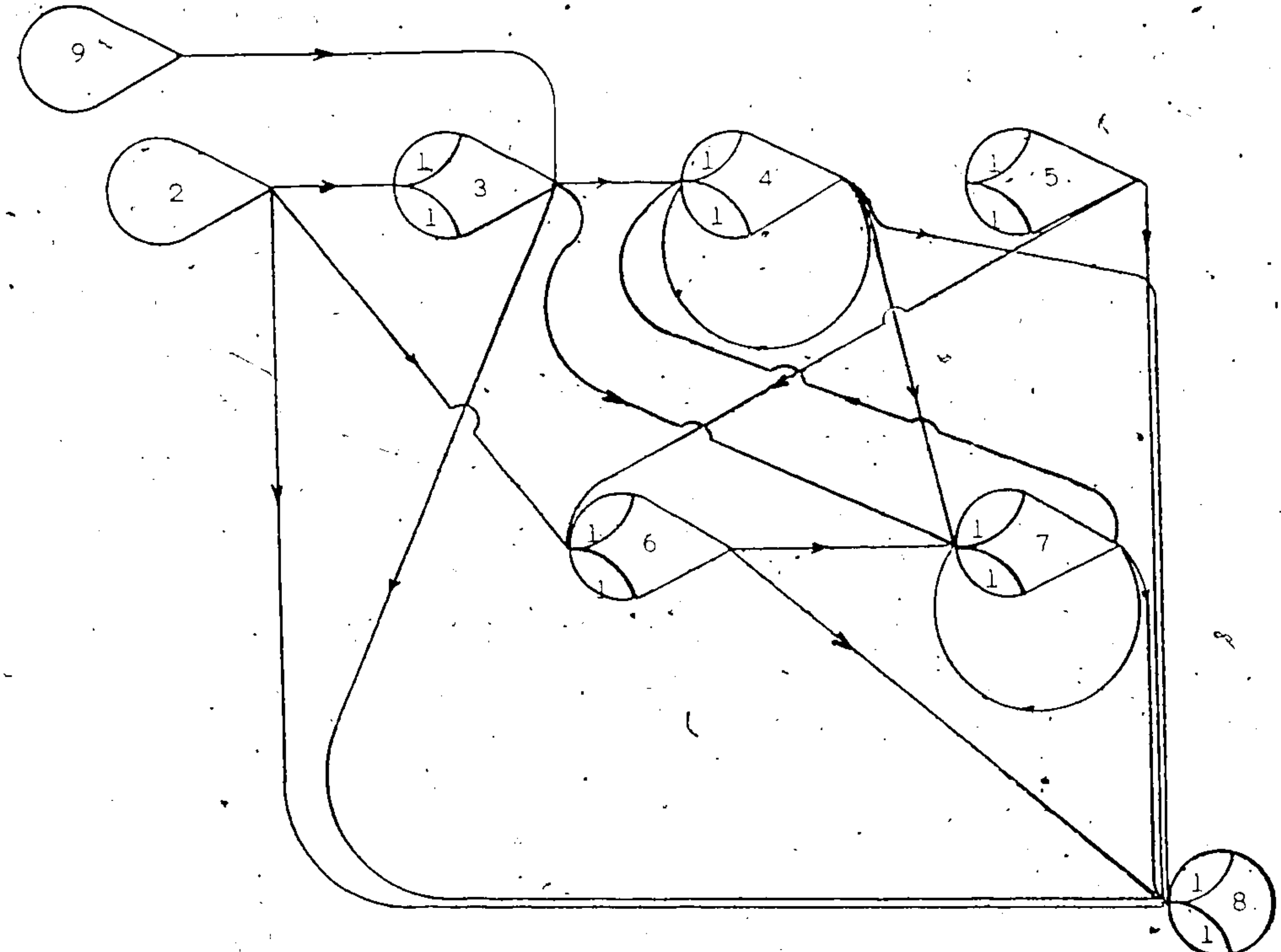

Fig.3(B) - GERT III elements representing a subscriber to The Detroit Evening News who is in state two.

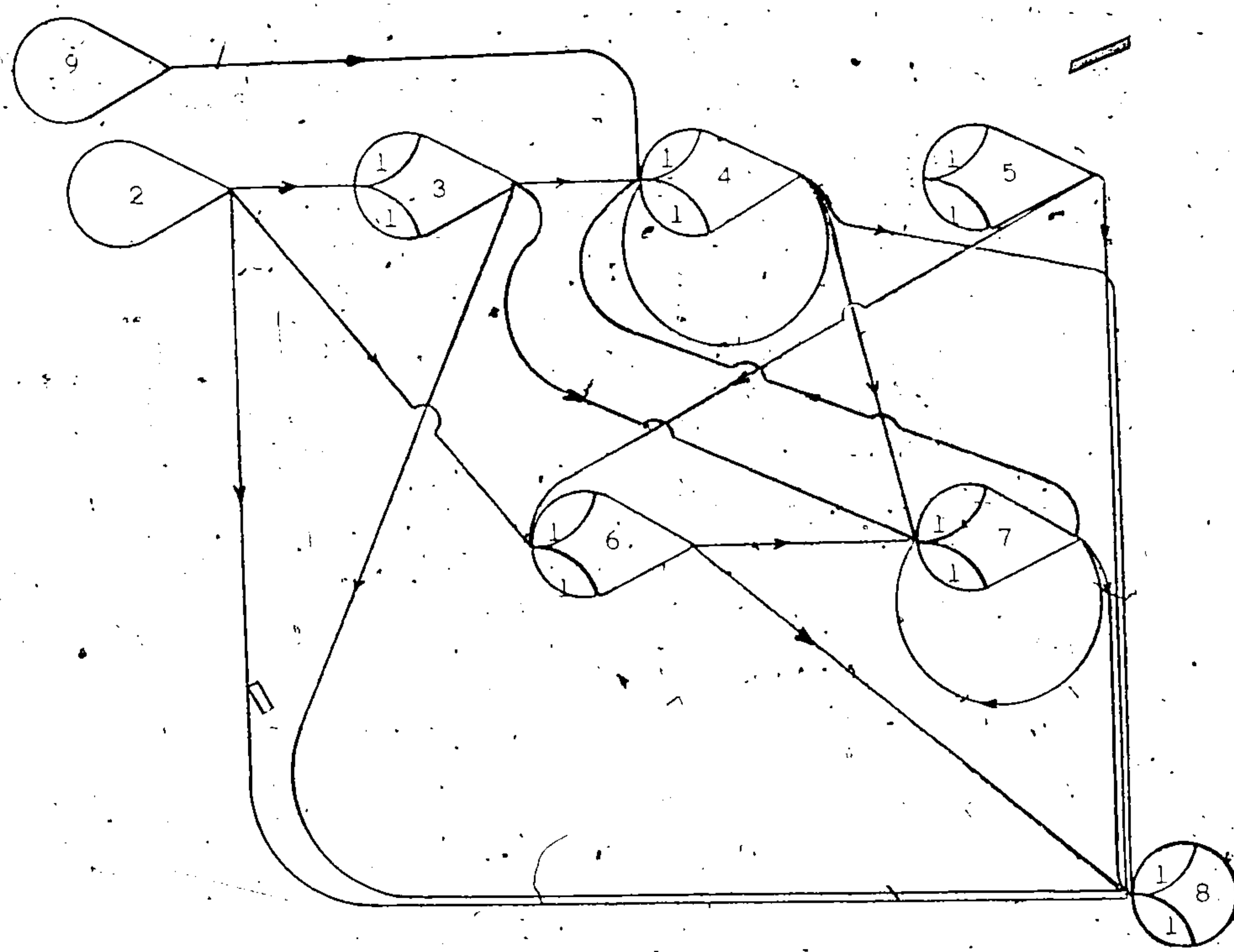

Fig.4(B) - GERT III elements representing a subscriber to The Detroit Evening News who is in state three.

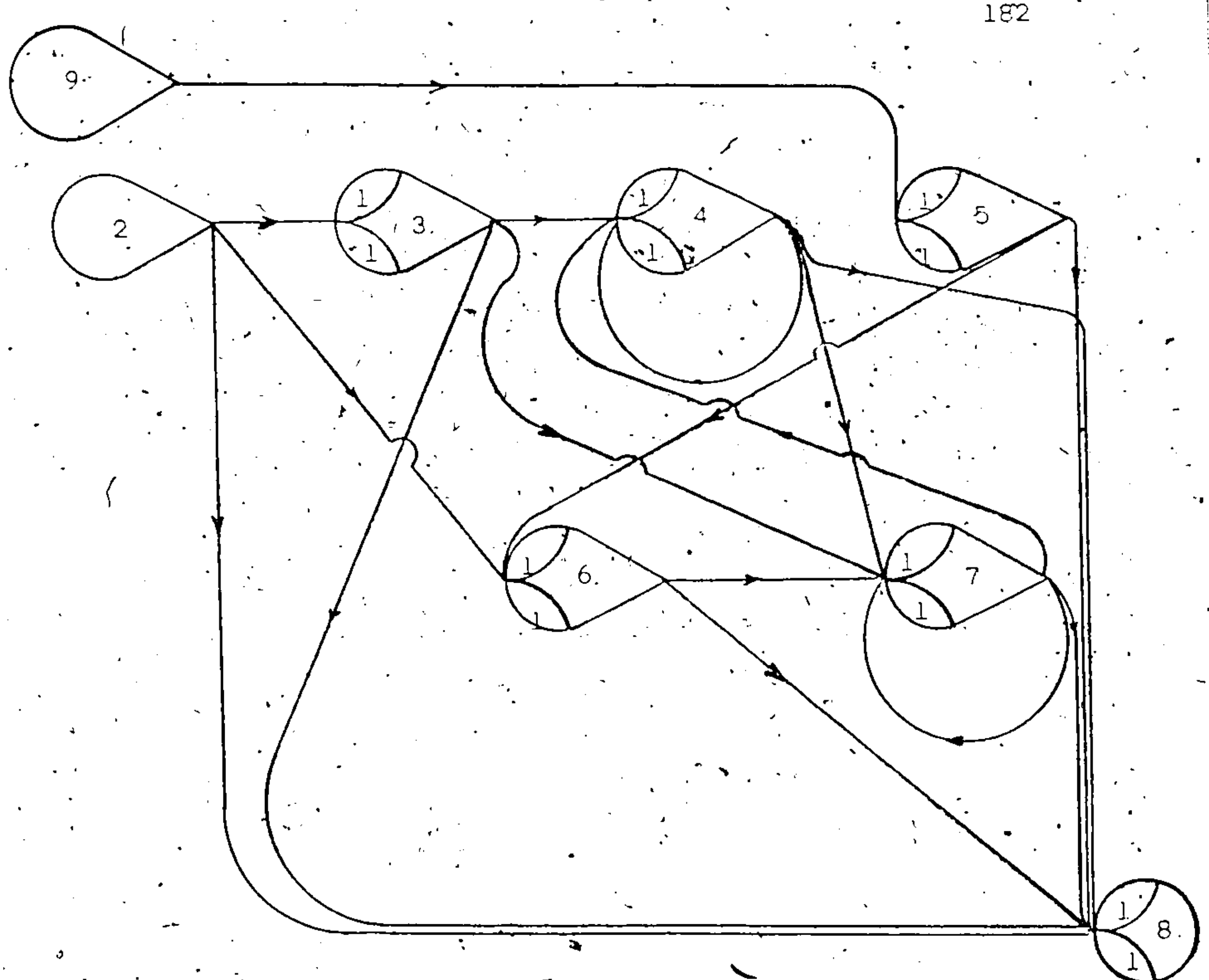

Fig.5(B) - GERT III elements representing a subscriber to The Detroit Evening News who is in state four.

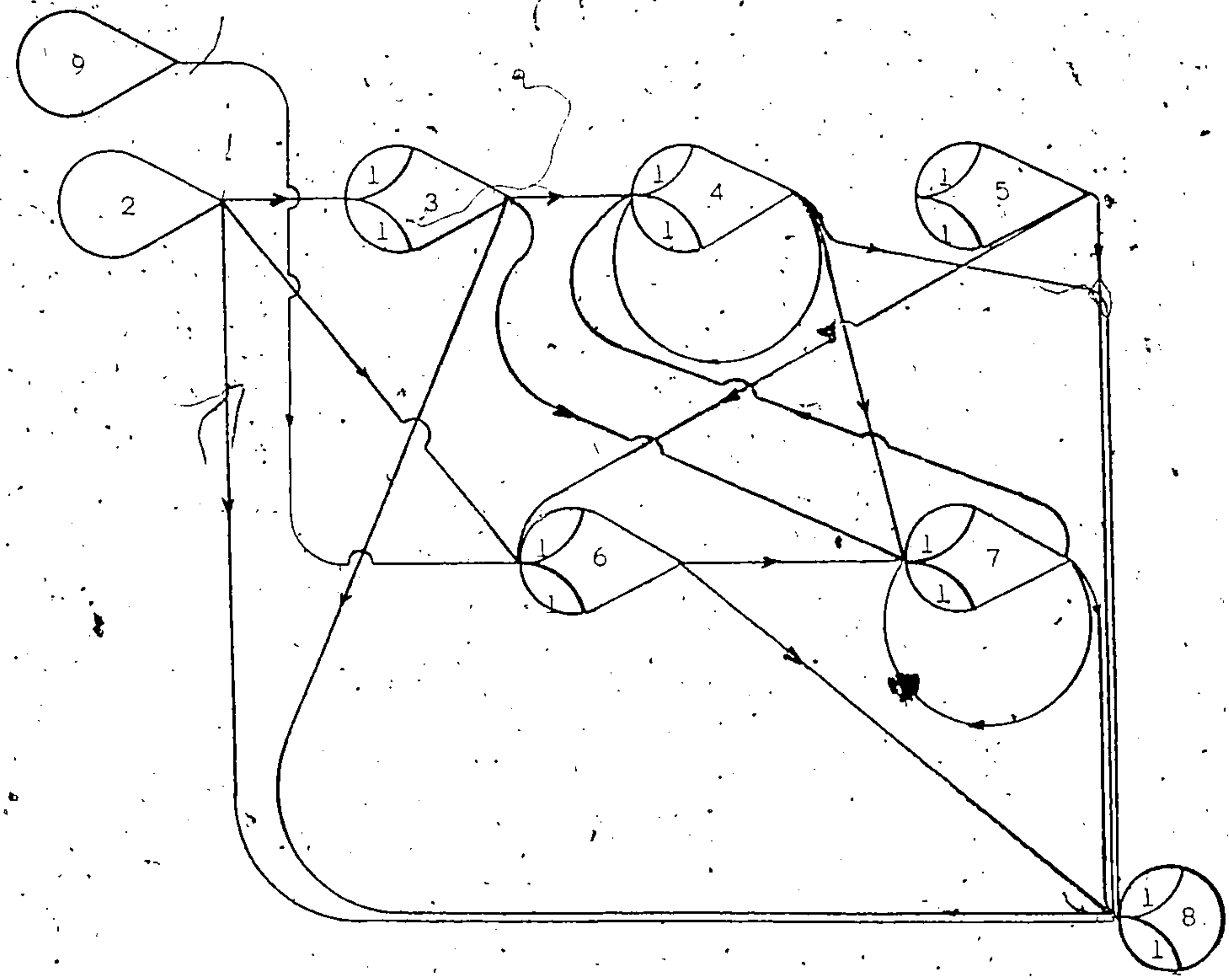

Fig. 6(B) - GERT III elements representing a subscriber to <u>The Detroit Evening News</u> who is in state five.

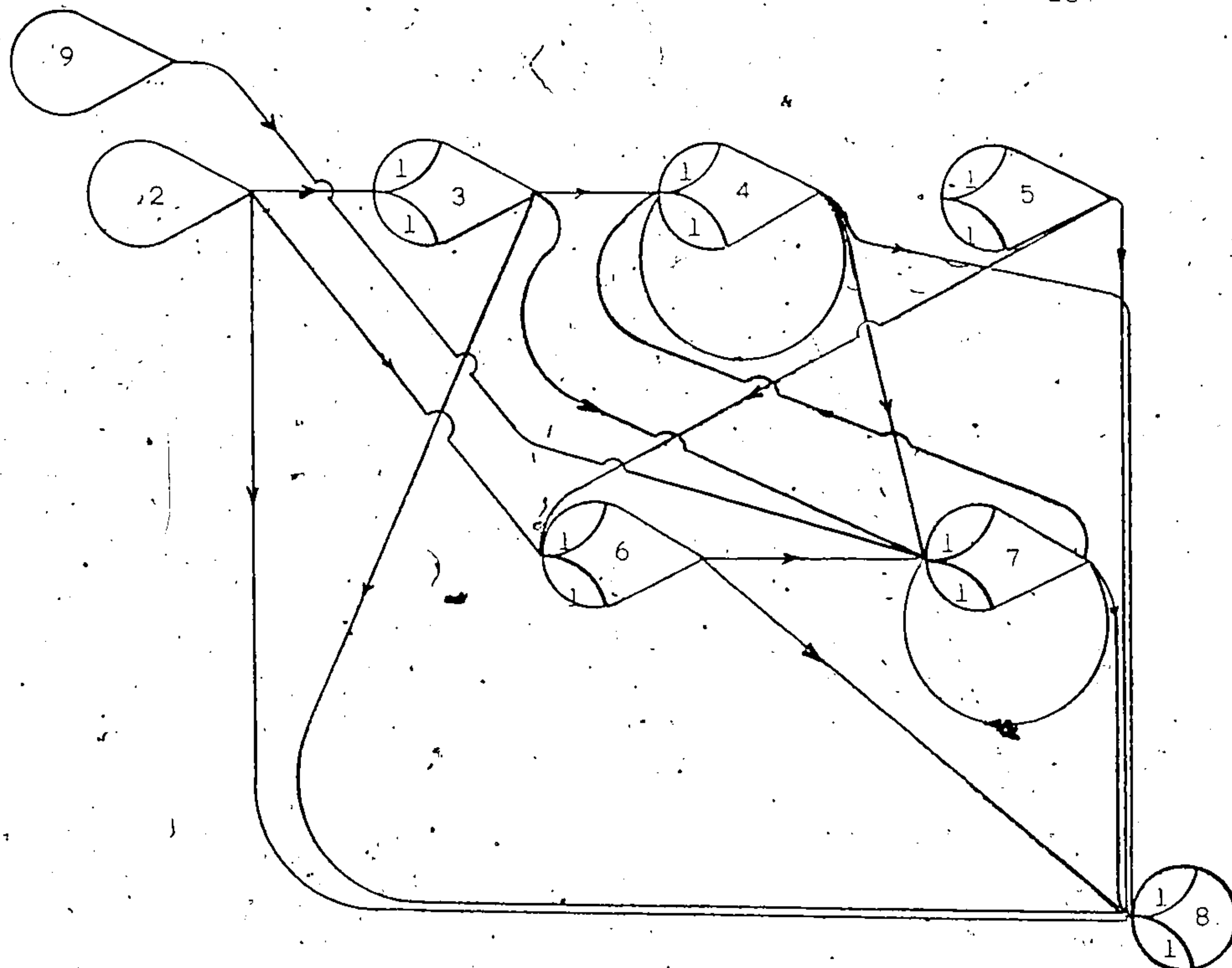

Fig.7(B). - GERT III elements representing a subscriber to <u>The Detroit Evening News</u> who is in state six.

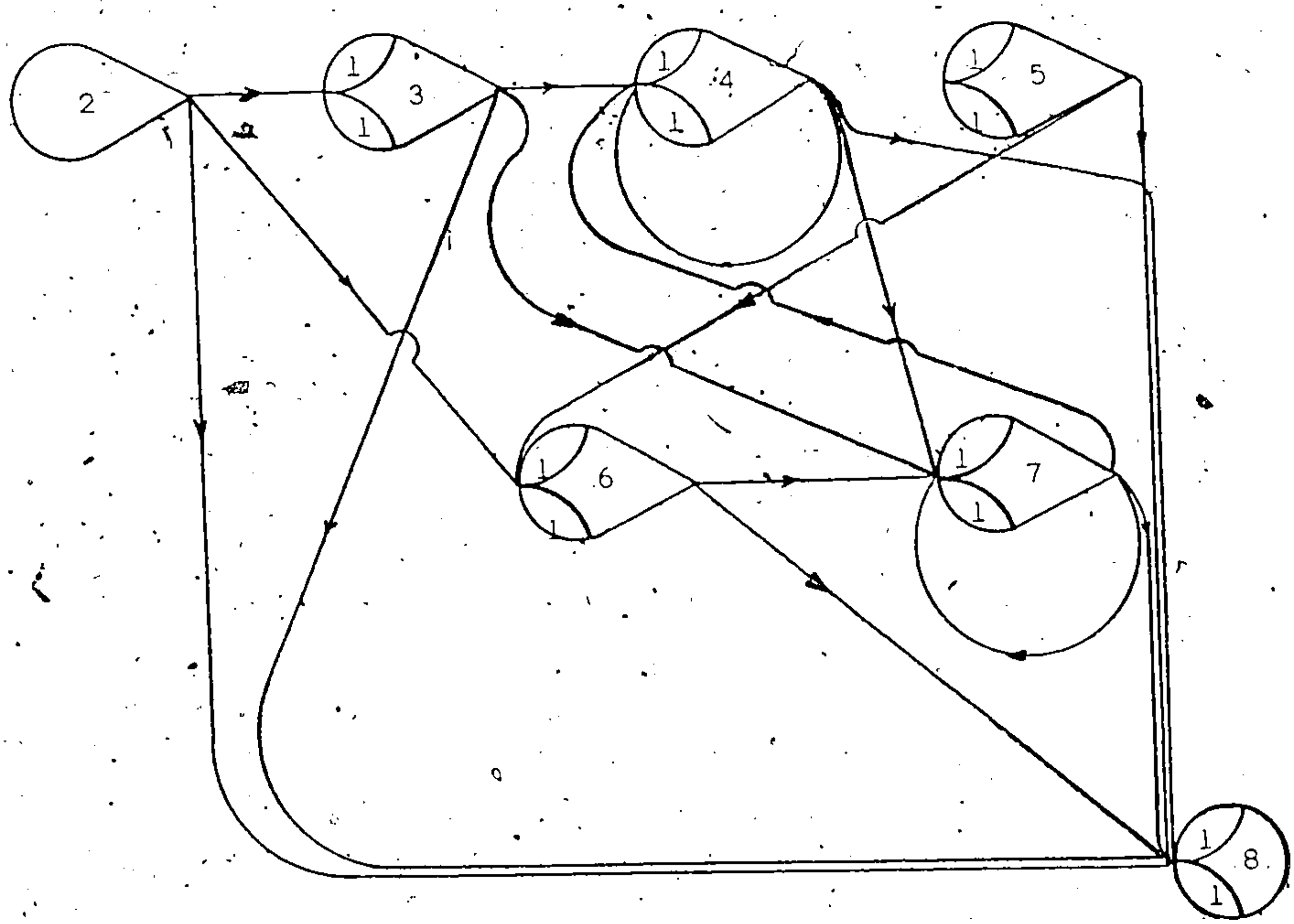

Fig.8(B) - GERT III elements representing a random subscriber to The Detroit Evening News.

APPENDIX (C)

TABLES

YEAR	POPULATION CENSUS (A)	NO. OF SUBSCRIBERS (B)	PROPORTION OF SUBSCRIBERS (A/B)	PROPORTION OF NON-SUBSCRIBERS (A/B)-1
1969	205002	57994	0.2828	0.7172
1970	206975	57881	0.2796	0.7204
1971	218646	57960	0.2650	0.7350
1972	218646	56924	0.2603	0.7397
1973	218646	58214	0.2662	0.7338

TABLE 1(C)a - Proportions of Subscribers and non-subscribers in the City Zone.

YEAR	TOTAL NO. OF HOUSE-HOLDERS (A)	NO. OF SUBSCRIBERS (B)	PROPORTION OF SUBSCRIBERS (A/B)	PROPORTION OF NON-SUBSCRIBERS (A/B)-1
1969	23614	19583	0.8292	0.1708
1970	30350	20083	0.6617	0.3383
1971	39645	20306	0.5121	0.4879
1972	39645	19654	0.4957	0.5043
1973	39645	20090	0.5067	0.4933

TABLE 1(C)b - Proportions of Subscribers and non-subscribers in the Retail Trading Zone.

YEAR	TOTAL NO.OF HOUSE-HOLDERS (A)	NO. OF SUBSCRIBERS (B)	PROPORTION OF SUBSCRIBERS (A/B)	PROPORTION OF NON-SUBSCRIBERS (A/B)-1
1969	20247	14577	0.7199	0.2801
1970	27500	15292	0.5560	0.4440
1971	24340	15725	0.6460	0.3540
1972	24340	15469	0.6355	0.3645
1973	24340	15951	0.6553	0.3447

TABLE 1(C)c - Proportions of Subscribers and non-subscribers in the County of Essex.

YEAR	TOTAL NO.OF HOUSE-HOLDERS (A)	NO. OF SUBSCRIBERS (B)	PROPORTION OF SUBSCRIBERS (A/B)	PROPORTION OF NON-SUBSCRIBERS (A/B)-1
1969	27028	9269	0.3429	0.6571
1970	28525	8451	0.2962	0.7038
1971	29665	7911	0.2665	0.7334
1972	29665	7140	0.2406	0.7594
1973	29665	6986	0.2354	0.7646

TABLE 1(C)d - Proportions of Subscribers and non-subscribers in the County of Kent.

YEAR	TOTAL NO.OF HOUSE-HOLDERS (A)	NO. OF SUBSCRIBERS (B)	PROPORTION OF SUBSCRIBERS (A/B)	PROPORTION OF NON-SUBSCRIBERS (A/B)-1
1969	29597	3714	0.1254	0.8746
1970	33300	3569	0.1071	0.8929
1971	33105	3293	0.0994	0.9006
1972	33105	2503	0.0756	0.9244
1973	33105	2033	0.0614	0.9386

TABLE 1(C)e - Proportions of Subscribers and non-subscribers in the County of Lambton.

YEAR	TOTAL NO.OF HOUSE-HOLDERS (A)	NO. OF SUBSCRIBERS (B)	PROPORTION OF SUBSCRIBERS (A/B)	PROPORTION OF NON-SUBSCRIBERS (A/B)-1
1969	71013	268	0.0037	0.9963
1970	82425	208	0.0025	0.9975
1971	85385	201	0.0023	0.9977
1972	85385	168	0.0019	0.9981
1973	85385	131	0.0015	0.9985

TABLE 1(C)f - Proportions of Subscribers and non-subscribers in the County of Middlesex.

YEAR	TOTAL NO. OF HOUSE-HOLDERS (A)	NO. OF SUBSCRIBERS (B)	PROPORTION OF SUBSCRIBERS (A/B)	PROPORTION OF NON-SUBSCRIBERS (A/B)-1
1969	228160	87141	0.3819	0.6181
1970	226163	86304	0.3816	0.6184
1971	276365	85965	0.3111	0.6889
1972	276365	82944	0.3001	0.6999
1973	276365	84056	0.3042	0.6958

TABLE 1(C)g - Proportions of Subscribers and non-subscribers for the total sales in southern Ontario.

K \ n	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0
2	0	0.612	0.051	0.041	0.053	0.027	0.025	0.025	0.017	0.020	0.016
3	0	0	0.372	0.080	0.047	0.050	0.051	0.047	0.017	0.036	0.028
4	0	0	0	0.239	0.053	0.061	0.067	0.060	0.041	0.045	0.041
5	0	0	0	0	0.159	0.046	0.043	0.050	0.046	0.039	0.054
6	0	0	0	0	0	0.098	0.036	0.036	0.033	0.037	0.042
7	0	0	0	0	0	0	0.087	0.028	0.032	0.027	0.030
8	0	0	0	0	0	0	0	0.046	0.023	0.030	0.021
9	0	0	0	0	0	0	0	0	0.036	0.012	0.018
10	0	0	0	0	0	0	0	0	0	0.018	0.015
11	0	0	0	0	0	0	0	0	0	0	0.011
$\sum_{K=0}^{\infty}\Phi_{11}(K/n)$	1	0.612	0.378	0.360	0.312	0.282	0.289	0.292	0.245	0.264	0.276

TABLE 2(C) - Values of $\Phi_{11}(K/n)$ for the City Zone, evaluated by GERT III simulation.

K \ n	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0
2	0	0.726	0.055	0.043	0.058	0.036	0.025	0.019	0.012	0.003	0.003
3	0	0	0.526	0.096	0.087	0.074	0.062	0.029	0.027	0.027	0.020
4	0	0	0	0.383	0.091	0.092	0.085	0.067	0.052	0.043	0.040
5	0	0	0	0	0.274	0.087	0.093	0.079	0.083	0.071	0.052
6	0	0	0	0	0	0.229	0.077	0.086	0.079	0.068	0.081
7	0	0	0	0	0	0	0.160	0.079	0.079	0.073	0.061
8	0	0	0	0	0	0	0	0.100	0.070	0.066	0.058
9	0	0	0	0	0	0	0	0	0.071	0.056	0.061
10	0	0	0	0	0	0	0	0	0	0.062	0.046
11	0	0	0	0	0	0	0	0	0	0	0.047
$\sum_{K=0}^{\infty} \Phi_{11}(K/n)$	1	0.726	0.581	0.522	0.510	0.518	0.502	0.459	0.473	0.469	0.469

TABLE 3(C) – Values of $\Phi_{11}(K/n)$ for the Retail Trading Zone, evaluated by GERT III simulation.

K \ n	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0
2	0	0.424	0.558	0.018	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0	0	0.168	0.504	0.289	0.020	0.001	0.0	0.0	0.0	0.0
4	0	0	0	0.082	0.305	0.406	0.195	0.011	0.001	0.0	0.0
5	0	0	0	0	0.031	0.166	0.354	0.307	0.123	0.019	0.0
6	0	0	0	0	0	0.012	0.086	0.256	0.330	0.233	0.073
7	0	0	0	0	0	0	0.005	0.042	0.164	0.269	0.294
8	0	0	0	0	0	0	0	0.003	0.015	0.090	0.221
9	0	0	0	0	0	0	0	0	0.0	0.013	0.051
10	0	0	0	0	0	0	0	0	0	0.0	0.006
11	0	0	0	0	0	0	0	0	0	0	0.001
$\sum_{K=0}^{\infty} \Phi_{11}(K/n)$	1	0.424	0.726	0.604	0.625	0.604	0.641	0.619	0.633	0.624	0.646

TABLE 4(C) - Values of $\Phi_{11}(K/n)$ for Essex County, evaluated by GERT III simulation.

K \ n	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0
2	0	0.709	0.026	0.015	0.019	0.025	0.018	0.014	0.015	0.012	0.007
3	0	0	0.501	0.030	0.035	0.033	0.027	0.020	0.015	0.025	0.019
4	0	0	0	0.354	0.039	0.028	0.026	0.035	0.031	0.021	0.034
5	0	0	0	0	0.243	0.039	0.030	0.033	0.049	0.032	0.027
6	0	0	0	0	0	0.193	0.028	0.028	0.029	0.031	0.028
7	0	0	0	0	0	0	0.110	0.026	0.021	0.034	0.024
8	0	0	0	0	0	0	0	0.090	0.023	0.016	0.021
9	0	0	0	0	0	0	0	0	0.060	0.030	0.222
10	0	0	0	0	0	0	0	0	0	0.043	0.017
11	0	0	0	0	0	0	0	0	0	0	0.032
$\sum_{K=0}^{\infty} \Phi_{11}(K/n)$	1	0.709	0.527	0.399	0.336	0.318	0.239	0.246	0.243	0.235	0.231

TABLE 5(C) - Values of $\Phi_{11}(K/n)$ for Kent County, evaluated by GERT III simulation.

n	0	1	2	3	4	5	6	7	8	9	10
P(n)	0.0	0.157	0.142	0.121	0.100	0.064	0.058	0.064	0.042	0.033	0.040
n	11	12	13	14	15	16	17	18	19	20	21
P(n)	0.028	0.022	0.021	0.015	0.012	0.011	0.08	0.012	0.008	0.009	0.003
n	22	23	24	25	26	27	28	29	30	Over 30	
P(n)	0.004	0.002	0.004	0.003	0.003	0.003	0.0	0.03	0.001	0.007	

TABLE 6(C) - Values of P(n) for the County of Lambton, evaluated by GERT III simulation.

n	0	1	2	3	4	5	6	7	8	9	10
P(n)	0	0.469	0.229	0.157	0.070	0.034	0.018	0.012	0.005	0.002	0.002

n	11	12	13	14	15	16	17	18	19	20	Over 20
P(n)	0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.001	0.0	0.0	0.0

TABLE 7(C) - Values of P(n) for the County of Middlesex evaluated by GERT III simulation.

K \ n	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0
2	0	0.713	0.029	0.027	0.023	0.024	0.029	0.020	0.017	0.021	0.009
3	0	0	0.481	0.057	0.042	0.038	0.041	0.032	0.017	0.040	0.024
4	0	0	0	0.347	0.055	0.051	0.044	0.050	0.046	0.031	0.049
5	0	0	0	0	0.234	0.050	0.044	0.067	0.043	0.038	0.045
6	0	0	0	0	0	0.170	0.048	0.032	0.036	0.040	0.039
7	0	0	0	0	0	0	0.131	0.034	0.029	0.045	0.037
8	0	0	0	0	0	0	0	0.098	0.037	0.027	0.035
9	0	0	0	0	0	0	0	0	0.058	0.031	0.013
10	0	0	0	0	0	0	0	0	0	0.040	0.016
11	0	0	0	0	0	0	0	0	0	0	0.033
$\sum_{K=0}^{\infty} \Phi_{11}(K/n)$	1	0.713	0.510	0.431	0.354	0.333	0.337	0.333	0.283	0.313	0.300

TABLE 8(C) - Values of $\Phi_{11}(K/n)$ for the total sales in southern Ontario, evaluated by GERT III simulation.

n	0	5	10	15	20	25	30	35	40	45
P(n)	0	0.465	0.150	0.097	0.083	0.057	0.041	0.028	0.017	0.019

n	50	55	60	65	70	75	80	85	90	95
P(n)	0.010	0.009	0.008	0.006	0.001	0.001	0.001	0.001	0.0	0.001

n	100	105	110	115	Over 115
P(n)	0.002	0.001	0.001	0.001	0.0

TABLE 9(C) - Values of P(n) for a subscriber to the Detroit Evening News who is in state one, evaluated by GERT III simulation.

n	0	5	10	15	20	25	30	35	40	45
P(n)	0	0.345	0.174	0.128	0.100	0.065	0.055	0.034	0.016	0.026

n	50	55	60	65	70	75	80	85	90	95
P(n)	0.014	0.011	0.010	0.010	0.001	0.002	0.001	0.0	0.002	0.001

n	100	105	110	115	Over 115
P(n)	0.001	0.002	0.001	0.001	0.0

TABLE 10(C) - Values of P(n) for a subscriber to The Detroit Evening News who is in state two, evaluated by GERT III simulation.

n	0	5	10	15	20	25	30	35	40	45
P(n)	0	0.274	0.196	0.139	0.106	0.073	0.063	0.018	0.031	0.015

n	50	55	60	65	70	75	80	85	90	95
P(n)	0.011	0.011	0.011	0.003	0.003	0.001	0.0	0.002	0.001	0.001

n	100	105	110	115	Over 115
P(n)	0.002	0.001	0.001	0.001	0.0

TABLE 11(C) - Values of P(n) for a subscriber to <u>The Detroit Evening News</u> who is in state three, evaluated by GERT III simulation.

n	0	5	10	15	20	25	30	35	40	45
P(n)	0	0.373	0.117	0.107	0.091	0.080	0.059	0.039	0.032	0.027

n	50	55	60	65	70	75	80	85	90	95
P(n)	0.016	0.013	0.009	0.010	0.010	0.004	0.002	0.003	0.001	0.0

n	100	105	110	115	Over 115
P(n)	0.001	0.003	0.001	0.001	0.0

TABLE 12(C). - Values of P(n) for a subscriber to <u>The Detroit Evening News</u> who is in state four, evaluated by GERT III simulation.

n	0	5	10	15	20	25	30	35	40	45
P(n)	0	0.300	0.131	0.120	0.105	0.089	0.069	0.044	0.030	0.030

n	50	55	60	65	70	75	80	85	90	95
P(n)	0.018	0.010	0.014	0.011	0.010	0.002	0.001	0.005	0.002	0.001

n	100	105	110	115	120	125	130	135	140	145	Over 145
P(n)	0.001	0.003	0.001	0.001	0.0	0.0	0.001	0.0	0.0	0.001	0.0

TABLE 13(C) - Values of P(n) for a subscriber to <u>The Detroit Evening News</u> who is in state five, evaluated by GERT III simulation.

n	0	5	10	15	20	25	30	35	40	45
P(n)	0	0.188	0.164	0.132	0.117	0.103	0.076	0.050	0.033	0.038

n	50	55	60	65	70	75	80	85	90	95
P(n)	0.027	0.010	0.015	0.011	0.012	0.003	0.002	0.005	0.003	0.002

n	100	105	110	115	120	125	130	135	140	145	Over 145
P(n)	0.002	0.003	0.001	0.0	0.0	0.0	0.001	0.001	0.0	0.001	0.0

TABLE 14(C) - Values of P(n) for a subscriber to _The Detroit Evening News_ who is in state six, evaluated by GERT III simulation.

n	0	5	10	15	20	25	30	35	40	45
P(n)	0	0.310	0.177	0.132	0.103	0.077	0.049	0.033	0.027	0.030

n	50	55	60	65	70	75	80	85	90	95
P(n)	0.014	0.016	0.007	0.009	0.003	0.003	0.002	0.001	0.002	0.0

n	100	105	110	115	120	125	130	135	140	145	Over 145
P(n)	0.001	0.001	0.001	0.001	0.0	0.0	0.0	0.0	0.0	0.001	0.0

TABLE 15(C) - Values of P(n) for a random subscriber to *The Detroit Evening News*, evaluated by GERT III simulation.

VITA AUCTORIS

1946	Born in Cairo, Egypt, on September 1st.
1962	Completed secondary education at El Noukrashy Secondary School.
1968	Graduated from Faculty of Engineering, Cairo University, Cairo, Egypt, in Industrial Engineering with degree of B.Sc.
1968	Worked as a research engineer at Menia Technical High Institute, Egypt.
1973	Joined University of Windsor for graduate studies.